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Effects of rotation and initial stress on peristaltic transport of fourth grade fluid with heat transfer and induced magnetic field

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ABSTRACT

This paper investigates the effect of rotation and initial stress on the peristaltic flow of an incompressible fourth grade fluid in asymmetric channel with magnetic field and heat transfer. Constitutive equations obeying the fourth grade fluid model are employed. Assumptions of long wavelength and low Reynolds number are used in deriving solution for the flow. Closed form expressions for the stream function, pressure gradient, temperature, magnetic force function, induced magnetic field and current density are developed. Pressure rise per wavelength and frictional forces on the channel walls have been computed numerically. Effects of rotation, initial stress and inclination of magnetic field on the axial velocity and pressure gradient are discussed in detail and shown graphically. Several limiting results can be obtained as the special cases of the problem under consideration. Numerical illustrations that show the physical effects and the pertinent features are investigated at the end of the paper.

1. Introduction

Peristaltic motion appears in a wide variety of physiological and engineering applications such as urine transport in the ureter, motion of spermatozoa in the cervical canal, the movement of chyme in the gastrointestinal tract, swallowing of food through esophagus, the vasomotion of small blood vessels, in roller and finger pumps and many others. Hayat and Noreen [1] investigated the peristaltic transport of fourth grade fluid with heat transfer and induced magnetic field. Wang et al. [2] discussed the peristaltic motion of a magneto-hydrodynamic micro-polar fluid in a tube. Hayat et al. [3] investigated the peristaltic transport of viscous fluid in a curved channel with compliant walls. Chia et al. [4] investigated the novel thermo-pneumatic peristaltic micro-pump with low temperature elevation on working fluid. Ali et al. [5] studied the non-Newtonian fluid flow induced by peristaltic waves in a curved channel. Pandey and Chaube [6] discussed the peristaltic transport of a viscoelastic fluid in a tube of non-uniform cross-section. Hayat et al. [7] investigated the influence of inclined magnetic field on peristaltic transport of fourth grade fluid in an inclined asymmetric channel. Tripathi et al. [8] investigated the peristaltic flow of viscoelastic fluid with the fractional Maxwell model through a channel. Tripathi et al. [9] studied the peristaltic transport of a generalized Burgers' fluid. Nadeem et al. [10] studied the influence of heat and mass transfer on peristaltic flow of a third order fluid in a diverging tube. Akbar et al. [11] investigated the peristaltic flow of a Williamson fluid in an inclined asymmetric channel with partial slip and heat transfer. Hayat et al. [12] discussed the peristaltic flow under the effects of an induced magnetic field and heat and mass transfer. Yldrm and Sezer [13] studied the effects of partial slip on the peristaltic flow of a MHD Newtonian fluid in an asymmetric channel. Nadeem et al. [14] studied the influence of heat transfer in peristalsis with variable viscosity. Ali and Hayat [15] investigated the peristaltic flow of a micro-polar fluid in an asymmetric channel. Srinivas and Kothandapani [16] discussed the influence of heat and mass transfer on MHD peristaltic flow through a porous space with compliant walls. Ali and et al. [17] discussed the peristaltic flow of a Maxwell fluid in a channel with compliant walls. Kothandapani and Srinivas [18] studied the staltic transport of a Jeffrey fluid under the effect of magnetic field in an asymmetric channel. Ali et al. [19] discussed the heat transfer analysis of peristaltic flow in a curved channel. Nadeem and Akbar [20] investigated the effects of heat transfer on the peristaltic transport of MHD Newtonian fluid with variable viscosity. Abd-Alla et al. [21] investigated the effect of the rotation, magnetic field and initial stress on peristaltic motion of micro-polar fluid. Mahmoud et al. [22] discussed the effect of the rotation on wave motion through cylindrical bore in a micro-polar porous medium. The dynamic behavior of a wet long bone that has been modeled as a piezoelectric hollow cylinder of crystal class 6 is investigated by Abd-Alla et al. [23].

The aim of the present paper is to discuss the peristaltic flow of fourth grade fluid through a two dimensional channel. The fluid is electrically conducted in the presence of a magnetic field with

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rotation and initial stress. The governing equations are modeled and then solved analytically under the consideration of long wavelength approximation. The pressure gradient, pressure rise, friction force, stream function, temperature and velocity are given and discussed. Numerical calculations are carried out and illustrated graphically in each case considered. The magnetic field, initial stress and rotation may be utilized as transport of bio-fluid in the ureters, intestines and arterioles. Comparison is made with existing results.

2. Formulation of the problem

Let us consider the peristaltic transport of an incompressible magneto-hydrodynamic (MHD) fourth grade fluid in a two dimensional channel of uniform thickness $2a$ subjected to initial stress, magnetic field and rotation. In Cartesian coordinate system we choose $\overline{X}$ in the direction of wave propagation and $\overline{Y}$ transverse to it. A constant magnetic field of strength H_0 acting in the transverse direction results in an induced magnetic field $H(\overline{h}_{\overline{x}}(\overline{X},\ \overline{Y},\ \overline{t}),\ \overline{h}_{\overline{y}}(\overline{X},\ \overline{Y},\ \overline{t}),\ 0)$. The total magnetic field is $H^+(\overline{h}_{\overline{x}}(\overline{X},\ \overline{Y},\ \overline{t}),\ H_0+\overline{h}_{\overline{y}}(\overline{X},\ \overline{Y},\ \overline{t}),\ 0)$.

The geometry of the channel wall is given by

$$\overline{h}(\overline{X},\ \overline{t}) = a+b\ \sin\left(\frac{2\pi}{\lambda}(\overline{X}-c\overline{t})\right) \tag{1}$$

where λ is the wavelength, a indicates the channel half width, b is the wave amplitude, c is the wave speed with which the infinite train of sinusoidal wave progresses along the wall in the positive $\overline{X}$ direction and $\overline{t}$ is the time. The equations governing the motion for the present problem are

$$\nabla \times V = 0, \tag{2}$$

$$\begin{aligned}\rho\frac{\partial V}{\partial t}+\rho\left[\overline{\Omega}\times(\overline{\Omega}\times V)+2\overline{\Omega}\times\frac{\partial V}{\partial t}\right] &= divT+\mu e(\nabla\times H^+)\times H^+ +\rho g\beta_T(T-T_0)\\ &= divT+\mu e\left[(\nabla\times H^+)H^+-\frac{\nabla H^{+2}}{2}\right]\\ &\quad+\rho g\beta_T(T-T_0)+p^*\nabla w_3,\end{aligned} \tag{3}$$

$$\rho C_p\frac{dT}{dt} = k\nabla^2T+Q_0, \tag{4}$$

$$\frac{dH^+}{d\overline{t}} = \nabla\times(V\times H^+)+\frac{1}{\varsigma}\nabla^2H^+ \tag{5}$$

where $\varsigma=\sigma\mu e$ is the magnetic diffusivity, C_P is the specific heat, T is the temperature, Q_0 is the constant of heat conduction and absorption and k is the thermal conductivity,

$$w_3 = \left(\frac{\partial\overline{v}}{\partial\overline{x}}-\frac{\partial\overline{u}}{\partial\overline{y}}\right)$$

The Cauchy ($\overline{T}$) (with pressure p, identity tensor $\overline{I}$ and an extra ($\overline{S}$) stress tensors) take the following relation:

$$\overline{T} = -p\overline{I}+\overline{S} \tag{6}$$

$$\begin{aligned}\overline{S} &= \mu\overline{A}_1+\alpha_1\overline{A}_2+\alpha_2\overline{A}_1^2+\beta_1'\overline{A}_3+\beta_2'(\overline{A}_2\overline{A}_1+\overline{A}_1\overline{A}_2)+\beta_3'(tr\overline{A}_2)\overline{A}_1+\gamma_1\overline{A}_4\\ &\quad+\gamma_2(\overline{A}_3\overline{A}_1+\overline{A}_1\overline{A}_3)+\gamma_3\overline{A}_2^2+\gamma_4(\overline{A}_2\overline{A}_1^2+\overline{A}_1^2\overline{A}_2)+\gamma_5(tr\overline{A}_2)\overline{A}_2\\ &\quad+\gamma_6(tr\overline{A}_2)\overline{A}_1^2+\{\gamma_7tr\overline{A}_3+\gamma_8tr(\overline{A}_2\overline{A}_1)\}\overline{A}_1\end{aligned} \tag{7}$$

$$\overline{A}_n = \frac{d\overline{A}_n}{d\overline{t}}+\overline{A}_{n-1}(grad\overline{V})+(grad\overline{V})^T\overline{A}_{n-1},\quad n>1 \tag{8}$$

$$\overline{A}_1 = (grad\overline{V})+(grad\overline{V})^T \tag{9}$$

where α_i $(i=1,\ 2)$, β_j' $(j=1-3)$ and γ_l $(l=1-8)$ are the material constants, $\overline{A}_n$ is the Rivilin–Ericksen tensors, $d/d\overline{t}$ is the material derivative, μ is the viscosity, tr is the trace, T in the superscript is the matrix transpose and the velocity $\overline{V}$ for two dimensional flows is

$$\overline{V} = [\overline{U}(\overline{X},\overline{Y},\overline{t}),\overline{V}(\overline{X},\overline{Y},\overline{t}),0]. \tag{10}$$

The Maxwell's relations in the absence of displacement current are

$$\nabla\times\mathbf{E} = 0,\quad \nabla\times\mathbf{H} = 0, \tag{11}$$

$$\nabla\times\mathbf{E} = -\mu e\frac{\partial H}{\partial t},\quad \nabla\times\mathbf{H} = \mathbf{J}, \tag{12}$$

$$\mathbf{J} = \sigma(\mathbf{E}+\mu_e(\mathbf{V}\times\mathbf{H})) \tag{13}$$

where $\mathbf{J}$, μ_e, σ, $\mathbf{E}$ and $\mathbf{H}$ denote the electric current density, the magnetic permeability, the electrical conductivity, the electric field and the magnetic field respectively.

Introducing a wave frame $(\overline{x},\ \overline{y})$ moving with velocity c away from the fixed frame $(\overline{X},\ \overline{Y})$ by the transformations

$$\left.\begin{array}{ll}\overline{x}=\overline{X}-c\overline{t}, & \overline{y}=\overline{Y}\\ \overline{u}(\overline{x},\overline{y})=\overline{U}-c, & \overline{v}(\overline{x},\overline{y})=\overline{V}\end{array}\right) \tag{14}$$

where $(\overline{U},\ \overline{V})$ and $(\overline{u},\ \overline{v})$ are the velocity components in the $\overline{X}$ direction of wave propagation and $\overline{Y}$ transverse to it in the fixed and moving coordinates, respectively; the fixed and wave frames respectively.

Taking into account the magnetic Lorentz force, initial stress, rotation and the energy and mass transfer, the equations governing MHD fluid are

$$\frac{\partial\overline{u}}{\partial\overline{x}}+\frac{\partial\overline{v}}{\partial\overline{y}} = 0, \tag{15}$$

$$\begin{aligned}\rho\left(\overline{u}\frac{\partial}{\partial\overline{x}}+\overline{v}\frac{\partial}{\partial\overline{y}}\right)\overline{u}+\frac{\partial\overline{p}}{\partial\overline{x}}-\rho\Omega\left[\Omega\overline{u}+2\frac{\partial\overline{v}}{\partial\overline{t}}\right] &= \frac{\partial\overline{S}_{xx}}{\partial\overline{x}}+\frac{\partial\overline{S}_{xy}}{\partial\overline{y}}-\frac{\mu_e}{2}\left(\frac{\partial H^{+2}}{\partial\overline{x}}\right)+\mu_e\left(\overline{h}_{\overline{x}}\frac{\partial\overline{h}_{\overline{x}}}{\partial\overline{x}}+\overline{h}_{\overline{y}}\frac{\partial\overline{h}_{\overline{x}}}{\partial\overline{y}}+H_0\frac{\partial\overline{h}_{\overline{x}}}{\partial\overline{y}}\right)\\ &\quad+P^*\frac{\partial}{\partial\overline{y}}\left(\frac{\partial\overline{v}}{\partial\overline{x}}-\frac{\partial\overline{u}}{\partial\overline{y}}\right)+\rho g\beta_T(T-T_0)\Big)\end{aligned} \tag{16}$$

$$\begin{aligned}\rho\left(\overline{u}\frac{\partial}{\partial\overline{x}}+\overline{v}\frac{\partial}{\partial\overline{y}}\right)\overline{v}+\frac{\partial\overline{p}}{\partial\overline{y}}-\rho\Omega\left[\Omega\overline{v}-2\frac{\partial\overline{u}}{\partial\overline{t}}\right] &= \frac{\partial\overline{S}_{xy}}{\partial\overline{x}}+\frac{\partial\overline{S}_{yy}}{\partial\overline{y}}-\frac{\mu_e}{2}\left(\frac{\partial H^{+2}}{\partial\overline{y}}\right)+\mu_e\left(\overline{h}_{\overline{x}}\frac{\partial\overline{h}_{\overline{y}}}{\partial\overline{x}}+\overline{h}_{\overline{y}}\frac{\partial\overline{h}_{\overline{y}}}{\partial\overline{y}}+H_0\frac{\partial\overline{h}_{\overline{y}}}{\partial\overline{y}}\right)\\ &\quad+P^*\frac{\partial}{\partial\overline{x}}\left(\frac{\partial\overline{v}}{\partial\overline{x}}-\frac{\partial\overline{u}}{\partial\overline{y}}\right)\Big)\end{aligned} \tag{17}$$

$$\rho C_p\left[\overline{u}\frac{\partial}{\partial\overline{x}}+\overline{v}\frac{\partial}{\partial\overline{y}}\right]\overline{T} = k\left[\frac{\partial^2\overline{T}}{\partial\overline{x}^2}+\frac{\partial^2\overline{T}}{\partial\overline{y}^2}\right]+Q_0. \tag{18}$$

We introduce the following non-dimensional quantities:

$$\begin{aligned}&x=\frac{\overline{x}}{\lambda},\quad y=\frac{\overline{y}}{a},\quad t=\frac{c\overline{t}}{\lambda},\quad p=\frac{a^2\overline{p}}{c\lambda\mu},\quad M^2=\mathrm{Re}S^2R_m,\quad \lambda_i=\frac{\alpha_ic}{\mu a}\,(i=1,\ 2),\\ &\delta=\frac{a}{\lambda},\quad S_{ij}=\frac{a\overline{S}_{ij}}{\mu c}\,(\text{for } i,\ j=1,\ 2,\ 3),\quad u=\frac{\overline{u}}{c},\quad v=\frac{\overline{v}}{c},\quad Re=\frac{ca\rho}{\mu},\\ &R_m=\sigma\mu_eac,\quad S=\frac{H_0}{c}\sqrt{\frac{\mu_e}{\rho}},\quad \phi=\frac{\overline{\phi}}{aH_0},\quad \eta_k=\frac{\gamma_kc^3}{\mu a^3}\,(k=1-8),\\ &\overline{h}_{\overline{x}}=\overline{\phi}_{\overline{y}},\quad \overline{h}_{\overline{y}}=-\overline{\phi}_{\overline{x}},\quad \mathbf{E}=\frac{-\overline{E}}{H_0ac\mu_e},\quad \mathrm{Pr}=\frac{\mu C_p}{k},\quad \gamma=\frac{\overline{T}-T_0}{T_0},\\ &Gr=\frac{\beta_TgT_0}{cv},\quad \beta_1=\frac{Q_0a_1^2}{kT_0},\quad \xi_j=\frac{\beta_j'c^2}{\mu a}\,(j=1,\ 2,\ 3).\end{aligned} \tag{19}$$

where Pr, δ, Re, R_m, S, P^*, Ω and M are the Prandtl number, the wave number, the Reynolds number, the magnetic Reynolds number, the Strommer number, the initial stress, the rotation and the Hartman