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Dynamic phase transitions and dynamic phase diagrams of the spin-2 Blume–Capel model under an oscillating magnetic field within the effective-field theory

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ABSTRACT

The dynamic phase transitions are studied in the kinetic spin-2 Blume–Capel model under a time-dependent oscillating magnetic field using the effective-field theory with correlations. The effective-field dynamic equation for the average magnetization is derived by employing the Glauber transition rates and the phases in the system are obtained by solving this dynamic equation. The nature (first- or second-order) of the dynamic phase transition is characterized by investigating the thermal behavior of the dynamic magnetization and the dynamic phase transition temperatures are obtained. The dynamic phase diagrams are constructed in the reduced temperature and magnetic field amplitude plane and are of seven fundamental types. Phase diagrams contain the paramagnetic (P), ferromagnetic-2 (F_2) and three coexistence or mixed phase regions, namely the F_2+P, F_1+P and F_2+F_1+P, which strongly depend on the crystal-field interaction (D) parameter. The system also exhibits the dynamic tricritical behavior.

1. Introduction

The Ising systems have been one of the most extensively studied systems in the statistical physics and the condensed-mater physics and have also been used as elementary models for variety of phenomena. This is so not only because of the relative simplicity with which approximate calculations for these models can be carried out and tested, but also because versions and extensions of models can be applied for description of a wide class of real systems. Although the majority of studies have focused on spin-1/2, spin-1 and spin-3/2 Ising models, higher spin systems are not without interest. One of the important higher spin systems is a spin-2 Ising model and has been paid much attention for many years. An early attempt to study the one-dimensional Ising model for $S=2$ (also $S=1$ and 3/2) was made by Obokata and Oguchi [1] by generalizing the Bethe approximation. They only calculated the energy and the specific heat exactly. Since then, various aspects of equilibrium properties of spin-2 Ising systems have been studied by well known methods in equilibrium statistical physics such as the mean-field approximation (MFA), the cluster variation method in pair approximation, the effective-field theory (EFT), the pair approximation with the discretized path integral representation and the four-spin model approximation [2]. The ground state phase diagrams of the systems have also been worked out [3]. The antiferromagnetic spin-2 Ising system was also studied on the Bethe lattice by the use of exact recursion relations [4]. We should also mention that the spins of Fe^{II} ions are spin-2 and it is experimentally found that these ions have anisotropy [5]. Moreover, the mixed spin (2, 5/2) Ising system is the prototypical system that has been used for studying magnetic behaviors of the molecular-based magnetic materials, such as $N(n\text{-}C_4H_9)_4Fe^{II}Fe^{III}(C_2O_4)_3$ [6,7] and $AFe^{II}Fe^{III}(C_2O_4)_3$ [$A=N(n\text{-}C_nH_{2n+1})_4$] [6,8,9].

Thus, although a lot is known about the equilibrium properties of the spin-2 Ising systems, the nonequlibrium properties of the model have not been as thoroughly explored. Recently, the dynamics of the spin-2 Ising systems under the presence of a time-dependent oscillating external magnetic field were studied by the dynamic MFA based on Glauber-type stochastic dynamics [10], especially the dynamic phase transition (DPT) temperatures were calculated and the dynamic phase diagrams are presented in the kinetic spin-2 Blume–Emery–Grifftihs model with repulsive biquadratic coupling [11,12]. In these works, it is found that spin-2 Ising systems have an interesting dynamic behavior and give rich dynamic phase diagrams within the dynamic MFA. This method is one of the oldest and important known methods, and

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it is still consistently used in the current literature. Moreover, it does offer a complete and relatively straightforward description of phase transitions and allows for complete studies of all thermodynamic properties in a uniform and relatively simple way. On the other hand, since this method neglects correlations between spins, it does not give accurate results, especially close to the critical point. It should be mentioned that there is a strong possibility that at least some of the first-order transition lines are very likely artifacts of the mean-field approach due to its limitations such as the correlation of spin fluctuations not being considered. Therefore, the dynamics of spin-2 Ising systems should be studied with more accurate techniques.

In this paper, we study the dynamical aspect of the spin-2 Ising model that contains a single-ion potential, which is known as the spin-2 Blume–Capel model, under a sinusoidal oscillating external magnetic field using the EFT with correlations based on the exact Van der Waerden identity. We employ the Glauber transitions rates [10] to construct the dynamic effective-field equation. We investigate time variations of the average magnetization to find the phases in the system. We also study the thermal behavior of the dynamic order parameters to characterize the nature (continuous and discontinuous) of the phase transitions and obtain the DPT points, and finally present the dynamic phase diagrams in (T/zJ, h/zJ) plane. We should also mention that the EFT method, without introducing mathematical complexity, can incorporate some effects of spin–spin correlations through the usage of the Van der Waerden identities and provide results that are quite superior to those obtained using the MFA. From this study, we also see the effect of spin correlations and artifacts of some of the first-order transition lines in the dynamic mean-field approach by comparing the results with the results given in Ref. [11]. We should also mention that some of our conclusions were presented at a conference [13] and summarized in the conference proceedings [14]. Finally, it is worthwhile mentioning that the EFT has been used to study dynamic phase transitions in the spin-1/2 Ising systems [15,16], recently.

This article is organized as follows. In Section 2, the spin-2 BC model is briefly described and the derivation of the dynamic effective-field equation is given using the Glauber-type stochastic dynamics in the presence of a time-dependent oscillating external magnetic field. Detailed numerical results and discussions are presented in Section 3, followed by a brief summary.

2. Model

The spin-2 Ising model that contains a single-ion potential or crystal field interaction in addition to the bilinear exchange interaction is known as the spin-2 Blume–Capel (BC) model, which is an extention of the spin-1 BC or simply the BC model [17]. The model is described by the following Hamiltonian:

$$H = -J_{ij}\sum_{\langle ij\rangle} S_i S_j - D\sum_i (S_i)^2 - h(t)\sum_i S_i, \tag{1}$$

where the S_i takes the value ± 2, ± 1 and 0 at each site i of a lattice and summation index $\langle ij\rangle$ denotes a summation over all pairs of the nearest neighbor sites. J_{ij} represents the spin–spin interaction strength between sites i and j. For simplicity all J_{ij} are taken equal to a constant $J>0$. D is the crystal-field interaction or a single-ion anisotropy constant, $h(t)$ is a time-dependent external oscillating magnetic field and is given by

$$h(t) = h_0 \sin(wt), \tag{2}$$

where h_0 and $w=2\pi v$ are the amplitude and the angular frequency of the oscillating field, respectively. The system is in contact with an isothermal heat bath at absolute temperature T.

Now, we use the effective-field theory with correlations to obtain the effective-field dynamic equation for the spin-2 Ising system. This method was first introduced by Honmura and Kaneyoshi [18] and Kaneyoshi et al. [19], which is a more advanced method dealing with Ising systems than the MFA, because it considers more correlations. Within the framework of the EFT, one finds that

$$\langle S_i^n\rangle = \left\langle \prod_{i=1}^{z}[1+A(\alpha)S_i+B(\alpha)(S_i)^2+C(\alpha)(S_i)^3+D(\alpha)(S_i)^4]\right\rangle f_n(x+h)\big|_{x=0}, \tag{3}$$

where $n=1$, 2, 3 and 4 for spin-2, $\alpha=J\nabla, \nabla=\partial/\partial x$ is a differential operator, z denotes the nearest-neighbor sites of the central site i and $z=4$ on the square lattice. The coefficients $A(\alpha)$, $B(\alpha)$, $C(\alpha)$ and $D(\alpha)$ for spin-2 in Eq. (3) are given by

$$\begin{aligned}
A(\alpha) &= \frac{1}{6}\left[8\sinh(\alpha)-\sinh(2\alpha)\right],\\
B(\alpha) &= \frac{1}{12}\left[16\cosh(\alpha)-\cosh(2\alpha)-15\right],\\
C(\alpha) &= \frac{1}{6}\left[\sinh(2\alpha)-2\sinh(\alpha)\right],\\
D(\alpha) &= \frac{1}{12}\left[\cosh(2\alpha)-4\cosh(\alpha)+3\right].
\end{aligned} \tag{4}$$

The functions $f_n(x+h)$ ($n=1$, 2, 3 and 4) for spin-2 are defined by

$$f_1(x+h) = \frac{1}{2}\frac{4\sinh[2\beta(x+h)]+2\sinh[\beta(x+h)]\exp(-3\beta D)}{\cosh[2\beta(x+h)]+\cosh[\beta(x+h)]\exp(-3\beta D)+\exp(-4\beta D)}, \tag{5a}$$

$$f_2(x+h) = \frac{1}{2}\frac{8\cosh[2\beta(x+h)]+2\cosh[\beta(x+h)]\exp(-3\beta D)}{\cosh[2\beta(x+h)]+\cosh[\beta(x+h)]\exp(-3\beta D)+\exp(-4\beta D)}, \tag{5b}$$

$$f_3(x+h) = \frac{1}{2}\frac{16\sinh[2\beta(x+h)]+2\sinh[\beta(x+h)]\exp(-3\beta D)}{\cosh[2\beta(x+h)]+\cosh[\beta(x+h)]\exp(-3\beta D)+\exp(-4\beta D)}, \tag{5c}$$

$$f_4(x+h) = \frac{1}{2}\frac{32\cosh[2\beta(x+h)]+2\cosh[\beta(x+h)]\exp(-3\beta D)}{\cosh[2\beta(x+h)]+\cosh[\beta(x+h)]\exp(-3\beta D)+\exp(-4\beta D)}. \tag{5d}$$

Eq. (3) is also exact and is valid for any lattice. If we try to exactly treat all the spin–spin correlations for that equation, the problem quickly becomes intractable. A first obvious attempt to deal with it is to ignore correlations; the decoupling approximation

$$\langle S_i S_{i'} \ldots S_{i^n}\rangle \cong \langle S_i\rangle\langle S_{i'}\rangle\ldots\langle S_{i^n}\rangle, \tag{6}$$

with $i\neq i'\neq\cdots\neq i^n$ has been introduced within the EFT with correlations [18,20,21]. In fact, the approximation corresponds essentially to the Zernike approximation [21] in the bulk problem, and has been successfully applied to a great number of magnetic systems, including the surface problems [18,20–22]. On the other hand, in the mean-field theory, all the correlations, including the self-correlations, are neglected. Based on this approximation, Eq. (3) is reduced to

$$m = \langle S_i\rangle = [1+A(\alpha)\langle S_i\rangle+B(\alpha)\langle S_i^2\rangle+C(\alpha)\langle S_i^3\rangle + D(\alpha)\langle S_i^4\rangle]^4 f_1(x+h)\big|_{x=0}, \tag{7}$$

$$q = \langle S_i^2\rangle = [1+A(\alpha)\langle S_i\rangle+B(\alpha)\langle S_i^2\rangle+C(\alpha)\langle S_i^3\rangle + D(\alpha)\langle S_i^4\rangle]^4 f_2(x+h)\big|_{x=0}, \tag{8}$$

$$r = \langle S_i^3\rangle = [1+A(\alpha)\langle S_i\rangle+B(\alpha)\langle S_i^2\rangle+C(\alpha)\langle S_i^3\rangle + D(\alpha)\langle S_i^4\rangle]^4 f_3(x+h)\big|_{x=0}, \tag{9}$$