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Ultrasound velocimetry of ferrofluid spin-up flow measurements using a spherical coil assembly to impose a uniform rotating magnetic field

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A R T I C L E I N F O

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A B S T R A C T

Ferrofluid spin-up flow is studied within a sphere subjected to a uniform rotating magnetic field from two surrounding spherical coils carrying sinusoidally varying currents at right angles and 90° phase difference. Ultrasound velocimetry measurements in a full sphere of ferrofluid shows no measureable flow. There is significant bulk flow in a partially filled sphere (1–14 mm/s) of ferrofluid or a finite height cylinder of ferrofluid with no cover (1–4 mm/s) placed in the spherical coil apparatus. The flow is due to free surface effects and the non-uniform magnetic field associated with the shape demagnetizing effects. Flow is also observed in the fully filled ferrofluid sphere (1–20 mm/s) when the field is made non-uniform by adding a permanent magnet or a DC or AC excited small solenoidal coil. This confirms that a non-uniform magnetic field or a non-uniform distribution of magnetization due to a non-uniform magnetic field are causes of spin-up flow in ferrofluids with no free surface, while tangential magnetic surface stress contributes to flow in the presence of a free surface.

Recent work has fitted velocity flow measurements of ferrofluid filled finite height cylinders with no free surface, subjected to uniform rotating magnetic fields, neglecting the container shape effects which cause non-uniform demagnetizing fields, and resulting in much larger non-physical effective values of spin viscosity $\eta' \sim 10^{-8}-10^{-12}$ N s than those obtained from theoretical spin diffusion analysis where $\eta' \leq 10^{-18}$ N s. COMSOL Multiphysics finite element computer simulations of spherical geometry in a uniform rotating magnetic field using non-physically large experimental fit values of spin viscosity $\eta' \sim 10^{-8}-10^{-12}$ N s with a zero spin-velocity boundary condition at the outer wall predicts measureable flow, while simulations setting spin viscosity to zero ($\eta'=0$) results in negligible flow, in agreement with the ultrasound velocimetry measurements. COMSOL simulations also confirm that a non-uniform rotating magnetic field or a uniform rotating magnetic field with a non-uniform distribution of magnetization due to an external magnet or a current carrying coil can drive a measureable flow in an infinitely long ferrofluid cylinder with zero spin viscosity ($\eta'=0$).

1. Introduction

Ferrofluid spin-up flow has often been experimentally studied using a finite height cylindrical container with and without a cover excited by an externally applied uniform rotating magnetic field. In our investigation we carefully examine various theories of ferrofluid spin-up flow in finite and infinite height cylinders and in spheres with experiments and computer simulations using COMSOL Multiphysics. This work shows that free surface magnetic stresses and the non-uniform demagnetization magnetic field that results from a finite height cylinder without a cover, even in an externally applied uniform rotating magnetic field, is the primary cause of this flow [1]. An infinitely long cylinder or a fully filled sphere of ferrofluid in a uniform rotating field, with zero spin viscosity ($\eta'=0$), has zero flow. We were not able to solve the ferrofluid flow equations for a ferrofluid filled sphere in a non-uniform rotating magnetic field using COMSOL; instead we solved the analogous problem assuming an infinitely long cylinder of ferrofluid in a non-uniform rotating magnetic field generated by an infinitely long permanent magnet of finite thickness, adjacent to the ferrofluid cylinder, in a uniform externally imposed rotating magnetic field [1].

2. Experiments

Our experiments involve using a ferrofluid in a spherical container surrounded by two concentric current carrying spherical coils with spatially orthogonal windings that generate perpendicular uniform magnetic fields. The two windings are driven by sinusoidal currents that are also out of phase by 90° in time as shown in Fig. 1, creating a uniform rotating magnetic field. A single

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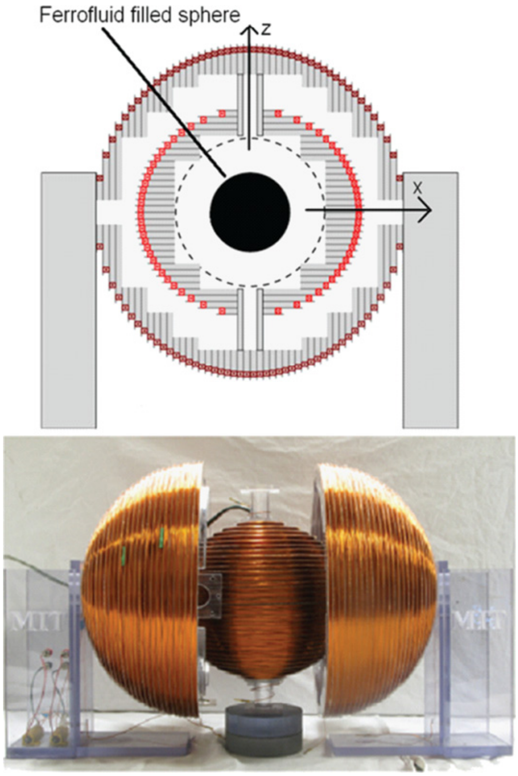

Fig. 1. Pair of space and time quadrature current coils wound on spherical Plexiglas structure used to generate uniform rotating magnetic field in centered ferrofluid filled sphere contained within inner spherical coil [1,2].

sphere of radius R filled with a ferrofluid, with an azimuthal surface current distribution at $r=R_1$ given as $K_\phi=(N_1 I\sin\theta/2R_1)$, where N_1 is the number of turns of the coil, I the spherical coil current in amperes and θ the polar angle, results in a uniform vertically oriented magnetic field inside the ferrofluid

$$H_z=\frac{N_1 I}{(3+\chi)R_1} \tag{1}$$

where χ is the magnetic susceptibility of the ferrofluid. The two spherical coils are of different sizes with the inner coil ($R_1=11.02$ cm and $N_1=1280$ turns) concentrically placed within the outer coil ($R_2=16.75$ cm and $N_2=1920$ turns). The differing radii of the two exciting coils are counterbalanced by the increase in the number of turns for the outer coil such that $(N_1/R_1)\approx(N_2/R_2)\approx 115$ turns/cm. Exciting the coils with the same magnitude current results in an $\approx 50\ (\text{Gauss/I})_{\text{rms}}$ uniform rotating field with a measured maximum of 2% spatial non-uniformity in the region within the smaller coil [2]. The spherical shape of the ferrofluid filled centered sphere ($R=5$ cm) inside both spherical coils guarantees spatially uniform demagnetizing factors of 1/3, which in the presence of a uniform rotating external magnetic field will ensure a uniform rotating magnetic field acting on the ferrofluid. An ultrasound velocimeter is used to measure the flow velocity along a line perpendicular to the velocimeter probe. The pulsed ultrasound technique uses reflections off of tracer latex particles (GrilTex-P1, average diameter 50 μm and density of 1.1 g/cm^3) suspended in the fluid flow to measure the flow velocity along the ultrasound beam [3–6]. The spherical coil assembly was allowed to return to room temperature between experiments ensuring that the experiments were all performed at a constant temperature. Experiments were performed using two Ferrotec ferrofluids, water-based MSGW11 and oil-based EFH1.

3. Theory

3.1. Ferrohydrodynamics

The fluid mechanics equations governing ferrohydrodynamics are conservation of linear and angular momentum equations [7]. The conservation of linear momentum equation assuming the flow is viscous dominated and incompressible ($\nabla\cdot\mathbf{v}=0$) is

$$0=-\nabla p+2\zeta\nabla\times\boldsymbol{\omega}+(\zeta+\eta)\nabla^2\mathbf{v}+\mu_0(\mathbf{M}\cdot\nabla)\mathbf{H} \tag{2}$$

The conservation of angular momentum neglecting inertial terms is

$$0=\mu_0\mathbf{M}\times\mathbf{H}+2\zeta(\nabla\times\mathbf{v}-2\boldsymbol{\omega})+(\eta'+\lambda')\nabla(\nabla\cdot\boldsymbol{\omega})+\eta'\nabla^2\boldsymbol{\omega} \tag{3}$$

where the variables are dynamic pressure p [N/m^2], fluid magnetization $\mathbf{M}$ [A/m], magnetic field $\mathbf{H}$ [A/m], spin velocity $\boldsymbol{\omega}$ [1/s], ferrofluid dynamic viscosity η [N s/m^2], vortex viscosity $\zeta=(3/2)\eta\phi_{vol}$ [N s/m^2] for small volume fraction ϕ_{vol} of magnetic nanoparticles [7,8], and λ' [N s] and η' [N s] are the bulk and shear coefficients of spin viscosity. Many planar and infinitely long cylindrical geometry problems have $\nabla\cdot\boldsymbol{\omega}=0$. For all COMSOL analyses in this paper we neglect the next to last term in Eq. (3) since we assume that η' [N s] and λ' [N s] are of the same order and their effect is small according to the spin diffusion theory.

3.2. Spin diffusion

Rosensweig [7] predicted a dimensional value of spin viscosity using a diffusion length model

$$\eta'\sim\eta l^2 \tag{4}$$

where l is the average distance between the solid particles given as

$$l=\left(\frac{4\pi}{3\phi_{vol}}\right)^{1/3}R \tag{5}$$

where R is the radius of the particle and ϕ_{vol} the volume fraction of the particles. The value of l and spin viscosity $\eta'$of Rosensweig's ferrofluid [7,9] with properties of $\eta=0.0012$ N s/m^2, $\phi_{vol}=0.012$, and $R=5\times10^{-9}$ m are then $l\sim35.2\times10^{-9}$ m and $\eta'\sim1.487\times10^{-18}$ N s. Rosensweig, using this value of spin viscosity, predicted an angular rotation rate that is a factor of 10^3–10^4 smaller than what was experimentally obtained [7]. Several authors have also mentioned this discrepancy [10–13]. Schumacher et al. [14] also separately derived a theoretical expression for spin viscosity, using a modified kinetic molecular theory of an ideal gas model, with ferrofluid properties $\eta=3.85\times10^{-3}$ Pa s, $\rho=1187.4$ kg/m^3, $\zeta=1.93\times10^{-3}$ Pa s, $R=12.5\times10^{-9}$ m (magnetic nanoparticle and surfactant), and $\phi_{vol}\sim0.334$, and determines a value of spin viscosity to be $\eta'=6.4\times10^{-20}$ N s while Eqs. (4) and (5) give $l\sim29\times10^{-9}$ m and $\eta'\sim3.25\times10^{-18}$ N s.

The theoretical determination of spin viscosity, using the spin diffusion theory, is many orders of magnitude smaller than the reported experimentally fit spin viscosity values [3,5,6]. For the ferrofluid EMG900_2 [3] with properties $\eta=4.5\times10^{-3}$ Pa s, $\rho=1030$ kg/m^3, $\zeta=2.9\times10^{-4}$ Pa s, $R=7\times10^{-9}$ m, and $\phi_{vol}=0.043$, the spin viscosity was estimated from fits to experiment to be $\eta'\approx10^{-8}$–10^{-12} N s while Eqs. (4) and (5) give $l\sim32.2\times10^{-9}$ m and $\eta'\sim4.67\times10^{-18}$ N s. Similarly for the water based ferrofluid MSGW11, with properties $\eta=2.02\times10^{-3}$ Pa s, $\rho=1200$ kg/m^3, $\zeta=0.83\times10^{-4}$ Pa s, $R=3.95\times10^{-9}$ m [15], and $\phi_{vol}=0.0275$, the spin viscosity was experimentally estimated to be $\eta'\approx10^{-8}$–10^{-9} N s [5,6] while Eqs. (4) and (5) give $l\sim21.1\times10^{-9}$ m and $\eta'\sim8.99\times10^{-19}$ N s. The oil-based ferrofluid EFH1 has properties $\eta=7.27\times10^{-3}$ Pa s, $\rho=1221$ kg/m^3, $\zeta=8.2\times10^{-4}$ Pa s, $R=5.3\times10^{-9}$ m [15], and $\phi_{vol}=0.0752$ with no reported experimentally fit values