Journal of Magnetism and Magnetic Materials 323 (2011) 1667–1670

Contents lists available at ScienceDirect

Journal of Magnetism and Magnetic Materials

journal homepage: www.elsevier.com/locate/jmmm

Theoretical analysis of magnetic sensor output voltage

Haishun Liu [a,b,*], Chaochao Dun [a], Linming Dou [b], Weiming Yang [a]

[a] School of Sciences, China University of Mining and Technology, Xuzhou 221116, China
[b] State Key Laboratory of Coal Resource and Mining Safety, China University of Mining and Technology, Xuzhou 221008, China

ARTICLE INFO

Article history:
Received 23 May 2010
Received in revised form
6 November 2010
Available online 4 February 2011

Keywords:
Stress measurement
Nondestructive evaluation
Output voltage
Principal stress
Magnetic method

ABSTRACT

The output voltage is an important parameter to determine the stress state in magnetic stress measurement, the relationship between the output voltage and the difference in the principal stresses was investigated by a comprehensive application of magnetic circuit theory, magnetization theory, stress analysis as well as the law of electromagnetic induction, and a corresponding quantitative equation was derived. It is drawn that the output voltage is proportional to the difference in the principal stresses, and related to the angle between the principal stress and the direction of the sensor. This investigation provides a theoretical basis for the principle stresses measurement by output voltage.

1. Introduction

Applied stress and residual stress may exert drastic effects on the performance of materials or lead to stress corrosion cracking, which results in eventual premature failure of components and structures. Thus, the measurement of stresses and the evaluation of their effects have always been the important topics in engineering, attracted considerable scientific and technological interests, and many efforts had been made in past decades to search for methods capable of measuring stresses accurately and quickly without damaging the materials to be tested. These so called nondestructive test methods, based on the variations in physical properties in materials with stress distribution, mainly involve radiographic and ultrasonic methods; among them, diffraction techniques, such as X-ray or neutron, are more popular, meanwhile, magnetic methods are more notable as well [1–4]. The authors' investigation focuses on the latter.

In former theoretical studies, the effects of stress on the microstructure and magnetization characteristics under uniaxial or biaxial stresses, such as hysteresis curve, susceptibility, coercivity, power loss, remnant magnetization, have been studied in detail [5–10]. Moreover, some successful theoretical investigations [11] had also been conducted in the view of the stress energy E_σ, based on an important relationship between E_σ and the stress σ [12], and the result is useful for discussing the effects of stress in terms of energy, which highlights the dependence of E_σ on θ, the angle between the stress and the magnetizing field, and λ_s, the saturation magnetostriction under the application of the stress. However, these studies are still more theoretical for the measurement of stress in structures.

In fact, the variation in a certain physical property with stress is usually transformed into detectable or visual parameters by instruments, generally, into voltage or current, which makes it more convenient to determine the stress state in structures. Thus, the relationship between the output voltage U and the stress σ is essential for determining the stress, and it was investigated by experiments (mainly by using calibration curves) in most former studies. However, theoretical analysis can seldom be seen, among them. Yamada et al. [4] had investigated the relationship under the application of the longitudinal stress, based on the assumption of elliptical distribution of directional permeability, and the results are effective under longitudinal stress. However, specimens are in complex stress state in most cases; meanwhile, the principal stresses and their directions are widely used to investigate the performance of structures; so a quantitative relationship between the principal stresses and the induced voltage is crucial for magnetic stress measurement, which makes it possible to evaluate the performance of structures, and it is absolutely necessary for further investigation [13].

2. Principle of magnetic stress measurement

One of the practical magnetic methods to measure stress is based on the magnetomechanical effect [14–16], which means that when a ferromagnetic material, such as steel, is applied by the stress, its magnetic property, e.g. permeability, would change. A sensor was made for stress measurement based on this effect,

---

* Corresponding author. Tel.: +86 139 1200 6872.
E-mail address: liuhaishun@126.com (H. Liu).

0304-8853/$ - see front matter © 2011 Elsevier B.V. All rights reserved.
doi:10.1016/j.jmmm.2011.01.034

which is composed of excitation poles, E_1 and E_2, and detection poles, D_1 and D_2, positioned perpendicular to each other, with coils wound around them (see Fig. 1) [17–18]. The research flowchart of this magnetic stress measurement instrument was shown in Fig. 2.

As a controlled current travels through the excitation coils, a magnetomotive force (mmf) in the magnetic circuit as well as a relating electromotive force (emf) in detection coils is introduced; meanwhile, the application of the stress to the specimen leads to different stress distributions as well as different permeabilities and different magnetic reluctances in different directions in the equivalent magnetic circuit, and results in different magnetic fluxes in different detection coils. So, induced voltages in each detection coil are different, and an output voltage related to the applied stress appears.

The induced emf as well as the output voltage can be investigated considering the similarity between the magnetic and electronic properties. Supposing R_1,R_2,R_3,R_4 are the corresponding magnetic reluctances between magnetic poles, E_1D_1, D_1E_2, D_2E_2, D_2E_1 through the magnetic core and specimen, r_1,r_2,r_3 and r_4 the corresponding magnetic reluctances of the air gap between magnetic poles E_1,E_2,D_1,D_2 and the specimen to be tested.

The physical parameters can be expressed as follows for the four-magnetic-pole sensor based on the magnetic circuit theory.

mmf

$$F_m = N_e I_e \tag{1}$$

and magnetic flux

$$\Phi_m = \frac{F_m}{R_m} = \frac{N_e I_e}{l/\mu A} \tag{2}$$

where N_e is the turn excitation coils, I_e the excitation current, R_m the magnetic reluctance, $R_m = l/\mu A$, μ the permeability of the material, l and A are the length and the section area of the magnetic circuit of the excitation pole, respectively, as shown in Fig. 3.

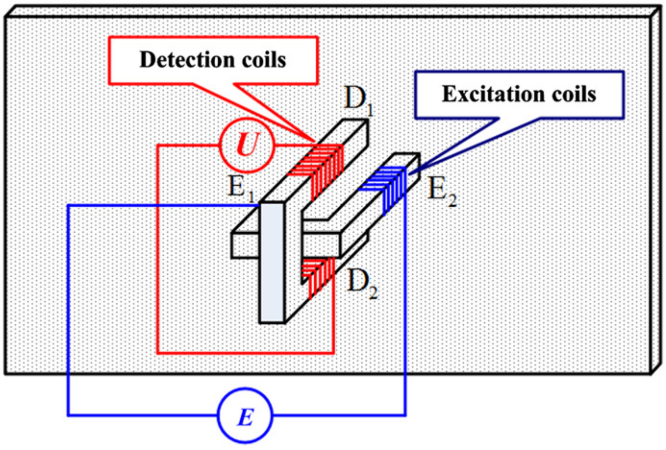

Fig. 1. Principle of magnetic sensor.

3. Theoretical analysis of the output voltage

3.1. Without stress

When there is no stress existing for the specimen, the magnetic reluctances in the projection area between the excitation poles and the detection ones are equal to each other, which means $R_1 = R_2 = R_3 = R_4$, the mmf induced in the magnetic poles D_1 and D_2 is exactly equal to each other, so the magnetic flux through the detection poles is equal to zero.

Here, the magnetic reluctance of the magnetic circuit can be expressed as

$$R = 2r_1 + R_e + R_m \tag{3}$$

where r_1 is the magnetic reluctance of the air gap between the excitation pole and the specimen, R_e and R_m are the magnetic reluctance in the core of excitation poles and that between the poles in specimen, respectively. The equivalent magnetic reluctance is given by $R_m = R_1R_5/R_1 + R_5$, based on the feature of parallel connectional magnetic circuit and the consideration of $R_1 = R_2 = R_3 = R_4$, with R_5 the magnetic reluctance between the excitation poles in the specimen.

Thus, the magnetic flux through the excitation poles is given by

$$\Phi_e = \frac{i_e N_e (R_1 + R_5)}{(2r_1 + R_e)(R_1 + R_5) + R_1 R_5} \tag{4}$$

Let $\phi_{e1},\phi_{e2},\phi_{e3}$, ϕ_{e4} be the magnetic fluxes through the magnetic reluctances R_1,R_2,R_3,R_4, as shown in Fig. 3, and the values of $\phi_{e1},\phi_{e2},\phi_{e3},\phi_{e4}$ are equal to each other considering the condition that $R_1 = R_2 = R_3 = R_4$. So the total magnetic flux in the circuit under this situation is

$$\Phi_e = 2\phi_{e1} + \phi_{e5} \tag{5}$$

An inverse relationship between the magnetic flux and the magnetic reluctance in the parallel connectional magnetic circuit leads to

$$\frac{\phi_{e1}}{\phi_{e5}} = \frac{R_5}{R_1} \tag{6}$$

thus

$$\phi_{e1} = \frac{\Phi_e R_5}{2(R_1 + R_5)} \tag{7}$$

Substitute Eqs. (1) and (4)–(7), we obtain

$$\phi_{e1} = \frac{i_e N_e R_5}{2(2r_1 + R_e)(R_1 + R_5) + R_1 R_5} \tag{8}$$

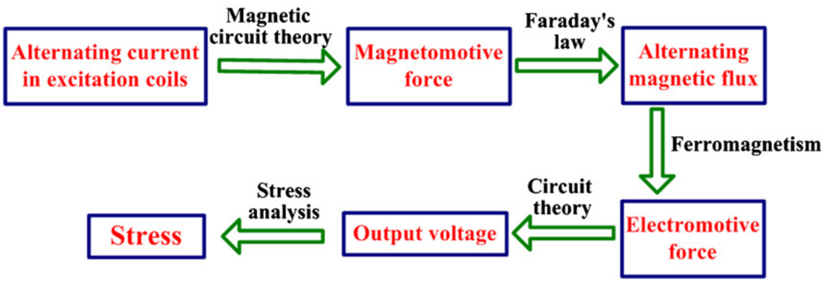

Fig. 2. Research flowchart of magnetic stress measurement.