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Magnetic susceptibility of MnZn and NiZn soft ferrites using Laplace transform and the Routh–Hurwitz criterion

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A B S T R A C T

This paper is devoted to study the Routh–Hurwitz stability criterion from the MnZn and NiZn soft ferrites using a phenomenological model with the gyromagnetic spin contribution and domain wall contribution. The magnetodynamic equation and the harmonic oscillator equation have been used to obtain the domain walls and the spin contribution of the magnetic susceptibility. The ferrite materials have been considered as linear, time invariant, isotropic and homogeneous, and the magnetization vector is proportional to the magnetic field vector. The resulting expression of the magnetization in time domain of both ferrites under study has been obtained by mean of the inverse Laplace transformation applying the residue method. The poles of the magnetic susceptibility have negative real parts, which ensures that the response decays exponentially to zero as the time increase. The degree of the numerator's polynomial of the magnetic susceptibility is less than the degree of denominator's polynomial in the magnetic susceptibility function: and the poles are located in the half left s-plane. Then the system is bounded-input, bounded-output (BIBO), and the results agree with the Routh–Hurwitz stability criterion for the MnZn and NiZn soft ferrites.

1. Introduction

The ferrite materials have been widely used as various electronic devices such as inductors, cable shielding, EMI suppression and/or electromagnetic wave absorbers in the relatively high-frequency region up to a few hundreds of MHz because of high magnetic permeability and high electrical resistivity. However, further improvements of the permeability performance in the higher frequency region up to a few GHz have been attempted [1]. The magnetic permeability of a material is a relevant factor to design devices, and it is advisable to investigate the behavior of the ferrite materials as a function of the frequency [2,3].

There are several phenomenological models of the magnetic dispersion and points out the common "denominator" of the oscillations of the mesoscopic domains corresponding to the magnetization walls and to the elastic media grains, respectively. The obtained results allow rather simple and accurate descriptions of the magnetic dispersion of ferrimagnetic materials, in the frequency domain of major interest for Electronics [4].

The ferrite media under study can be considered as linear, time invariant, isotropic and homogeneous. The magnetization vector $\vec{M}$ is proportional to the magnetic field vector $\vec{H}$, this can be expressed as [5,6]

$$\vec{M}(t) = (\chi * \vec{H})(t) \tag{1}$$

where t is the time in seconds and χ is the magnetic susceptibility.

Under time harmonic conditions the convolution type equation (1) gives rise to the relationship between Fourier transforms such as [6]

$$\vec{M}(\omega) = \chi(\omega)\vec{H}(\omega) \tag{2}$$

When a electromagnetic field is applied to a magnetic material, a dissipation of heat will be produced on it, and these magnetic losses are usually expressed by means of the imaginary part of the complex susceptibility of the ferrite as [7]

$$\chi(\omega) = \chi'(\omega) - j\chi''(\omega) \tag{3}$$

where $\chi'(\omega)$ and $\chi''(\omega)$ are related by means of the Kramers–Kronig equations [3,7].

In the frequency range from RF to microwaves, the magnetic susceptibility spectra of the ferrite can be characterized by the different magnetizing mechanisms, domain wall motion and gyromagnetic spin rotation [8]. So, magnetic susceptibility χ can be expressed as the contribution of two terms, gyromagnetic spin χ_s and domain wall χ_d [9].

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Applying the Laplace transform to the magnetization vector $\overrightarrow{M}(t)$ of (1) as

$$\overrightarrow{M}(s) = \chi(s)\overrightarrow{H}(s) \tag{4}$$

where $\chi(s) = \chi(s)' - j\chi(s)''$ is the complex magnetic susceptibility in the Laplace domain.

2. Theory

2.1. Domain wall contribution

Domain wall process can be studied with an equation of motion in which the pressure kH is equated to the sum of the three terms [2,3]:

$$m_e \frac{d^2 z}{dt^2} + \beta \frac{dz}{dt} + \delta z = kH \tag{5}$$

where m_e is the effective mass, β is the damping factor, and δ the elasticity factor, while k is a proportionality factor.

Assuming a Laplace variation of the magnetic field $H = H_0 e^{st}$ and the displacement $z = z_0 e^{st}$:

$$m_e s^2 z(s) + \beta s z(s) + \delta z(s) = kH(s) \tag{6}$$

The magnetization vector of N particles [3] can be expressed as

$$M(s) = Npz(s) \tag{7}$$

where p is the intensity of magnetic pole.

From (6) and (7):

$$M(s) = \frac{NpkH(s)}{m_e s^2 + \beta s + \delta} \tag{8}$$

From (4) and (8), the contribution of the domain wall to the magnetic susceptibility $\chi_d(s)$ can be written as

$$\chi_d(s) = \frac{\omega_d^2 \chi_{d0}}{s^2 + \beta_1 s + \omega_d^2} \tag{9}$$

where $\omega_d^2 = \delta/m$ is the resonance frequency of domain wall, $\beta_1 = \beta/m_e$ is the equivalent dumping factor and $\chi_{d0} = kpN/\delta$.

2.2. Spin contribution

Gyromagnetic spin contribution can be studied with a magnetodynamics equation [2,3]:

$$\frac{d\overrightarrow{M}}{dt} = \gamma_e \overrightarrow{M} \times \overrightarrow{H} + \frac{\alpha}{|\overrightarrow{M}|} \overrightarrow{M} \times \frac{d\overrightarrow{M}}{dt} \tag{10}$$

where γ_e is the gyromagnetic ratio and α is the damping factor.

Assuming an excitation:

$$H = H_i + he^{st} \tag{11}$$

$$M = M_0 + me^{st} \tag{12}$$

where H_i is the total internal field and M_0 is the saturation magnetization of the ferrite and me^{st} and he^{st} are the dynamic parts of the fields.

The magnetic susceptibility χ_s can be expressed as

$$\chi_s = \frac{(\omega_s + s\alpha)\omega_s \chi_{s0}}{s^2(1+\alpha^2) + 2\omega_s \alpha s + \omega_s^2} \tag{13}$$

where $\omega_s = -\gamma_e H_i$ is the resonance frequency of spin component and $\chi_{s0} = -\gamma M_0$ is the static magnetic susceptibility.

2.3. Total susceptibility

The total magnetic susceptibility $\chi(s)$ of the soft ferrite can be computed as the contribution of $\chi_d(s)$ and $\chi_s(s)$ component [10]:

$$\chi(s) = \frac{s^3 a + s^2 b + cs + d}{(s^2 + \beta_1 s + \omega_d^2)\left(s^2 + s\frac{2\omega_s \alpha}{(1+\alpha^2)} + \frac{\omega_s^2}{(1+\alpha^2)}\right)} \tag{14}$$

where

$$a = \frac{(\alpha \omega_s \chi_{s0})}{1+\alpha^2}$$

$$b = \frac{(1+\alpha^2)\omega_d^2 \chi_{d0} + \chi_{s0}\omega_s^2 + \alpha \omega_s \chi_{s0} \beta_1}{1+\alpha^2}$$

$$c = \frac{2\omega_s \alpha \omega_d^2 \chi_{d0} + \chi_{s0}\omega_s^2 \beta_1 + \alpha \omega_s \chi_{s0} \omega_d^2}{(1+\alpha^2)}$$

$$d = \frac{\omega_s^2 \omega_d^2 \chi_{d0} + \chi_{s0}\omega_s^2 \omega_d^2}{(1+\alpha^2)} \tag{15}$$

3. Material and methods

The concept of stability is extremely important in system studies. This is true for the simple reason that stability is, in almost all cases, the minimum condition that a system must meet in order to behave acceptably [11]. On the other hand, it does not follow that a system will exhibit acceptable behaviour just because it is stable. The notion of stability involves the behaviour of the system when it is subjected to general inputs. If the system output is bounded for all time for every bounded input, the system is said to be bounded-input, bounded-output (BIBO). First, requiring the system with a transfer function $\chi(s) = N(s)/D(s)$ to be BIBO, the degree of $N(s)$ must be less than or equal to the degree of $D(s)$ [11]. If any system pole lies in the right hand s-plane the system response will grow without bound as a result of any finite initial condition on the output of its derivatives [12,13]. Therefore, BIBO stability requires strictly left hand plane poles [11].

The Routh–Hurwitz criterion is a method for determining the presence the number of roots of a polynomial with positive real parts [12,14].

From (14), the poles of the magnetic susceptibility function $\chi(s)$, can be expressed as (Fig. 1):

$$S_{1,2} = \frac{-\beta_1 \pm \sqrt{\beta_1^2 - 4\omega_d^2}}{2} \tag{16}$$

$$S_{3,4} = \frac{-\frac{2\omega_s \alpha}{(1+\alpha^2)} \pm \sqrt{\left(\frac{2\omega_s \alpha}{1+\alpha^2}\right)^2 - 4\frac{\omega_s^2}{1+\alpha^2}}}{2} \tag{17}$$

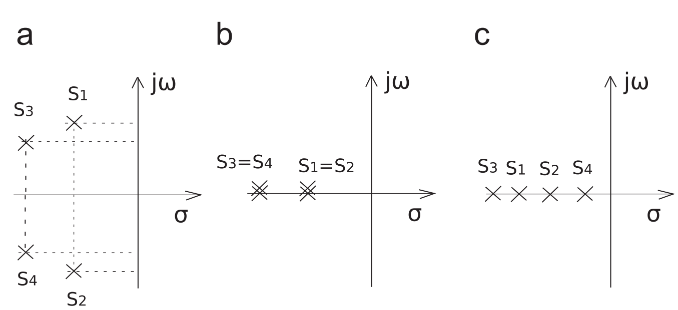

Fig. 1. Allowed locations of the poles of the magnetic susceptibility function.