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# Magnetic properties of the mixed ferrimagnetic ternary system with a single-ion anisotropy on the Bethe lattice

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ABSTRACT

The magnetic properties of the ternary system ABC consisting of spins $\sigma = \frac{1}{2}$, $S = 1$, and $m = \frac{3}{2}$ are investigated on the Bethe lattice by using the exact recursion relations. We consider both ferromagnetic and antiferromagnetic exchange interactions. The exact expressions for magnetizations and magnetic susceptibilities are found, and thermal behaviors of magnetizations and susceptibilities are studied. We construct the phase diagrams and find that the system exhibits one, two or even three compensation temperatures depending on the values of the interaction parameters in the Hamiltonian. Moreover, the system undergoes a second-order phase transition for the coordination number $q \leqslant 3$ and a second- and first-order phase transitions for $q > 3$; hence the system gives a tricritical point. The system also exhibits the reentrant behaviors.



## 1. Introduction

The magnetic properties of molecular-based magnetic materials have been studied actively for the last two decades. Among these compounds, a particular interest has been paid to Prussian blue analogs that offer advantages over the other magnets, such as photo-induced magnetism [1–3], anisotropic photo-induced magnetism in thin films [4,5], a charge-transfer-induced spin transitions [6–10], and hydrogen storage capacity [11]. Moreover, these materials may exhibit at most two compensation points [12]. The compensation point is a special point that appears below the critical temperature, at which the total magnetizations vanish below its transition temperatures. The existence of a compensation temperature is an interesting phenomenon with important technological applications in the field of thermomagnetic recording. Many more experimental works have been done about Prussian blue analogs (see Refs. [13–19], and references therein).

On the other hand, the ternary alloy $AB_pC_{1-p}$ composed of Prussian blue analogs has also been studied theoretically. For example, the magnetic properties of a mixed ferro–ferrimagnetic ternary alloy of the type $AB_pC_{1-p}$ consisting of three different metal ions with Ising spins $\frac{1}{2}$, 1, $\frac{3}{2}$ [20–22]; 1, $\frac{3}{2}$, $\frac{5}{2}$ [23–28]; $\frac{3}{2}$, 2, $\frac{5}{2}$ [29–31], $\frac{1}{2}$, 1, $\frac{5}{2}$ [32], and 0, $\frac{1}{2}$, $\frac{3}{2}$ [33] have been investigated, theoretically. These studies were done within the effective-field theory (EFT) [20,22,27], the Monte Carlo (MC) simulation [28,32], the mean-field theory (MFT) based on Bogoliubov inequality for the Gibbs free energy [21,24,29] and the MFT [23,25,26,30,31,33]. The ground-state phase diagrams of the $AB_pC_{1-p}$ ternary alloy consisting of Ising spins $\frac{3}{2}$, 1, and $\frac{5}{2}$ in the presence of single-ion anisotropy have also been constructed [34]. We have very recently studied the critical behavior of the mixed Ising model of the type ABC ternary alloy consisting of spins $\sigma = \frac{1}{2}$, $S = 1$, and $m = \frac{3}{2}$ on the Bethe lattice by using the exact recursion equations [35]. Our investigations have been done for the simplest case in which the bilinear exchange interactions are taken to be the same and only ferromagnetic exchange interactions for all interactions between Ising spins are considered. Thus two simple phase diagrams, in which only ferrimagnetic and paramagnetic phases occur, were obtained. We have also found that the system always undergoes a second-order phase transition for $q \leqslant 3$, $q$ is the coordination number; and it undergoes a second- and first-order phase transitions for $q > 3$. Hence the system exhibits tricritical point. Moreover, the compensation temperatures do not appear in the system for ferromagnetic exchange interactions.

The purpose of the present paper is to study the magnetic properties of a mixed ferrimagnetic ternary system of the type

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ABC consisting of spins $\sigma=\frac{1}{2}$, $S=1$, and $m=\frac{3}{2}$ on the Bethe lattice by using the exact recursion relations in detail, such as magnetizations, transition temperatures, and compensation temperatures. Since multicritical behavior of the system strongly depends on the interactions between the spins, we take different bilinear nearest-neighbor interactions between Ising spins. We also consider both ferromagnetic and anti-ferromagnetic exchange interactions. These considerations lead us to study the magnetic properties of the ABC ternary system corresponding to the Prussian blue analogs in detail. The appearance of compensation temperatures is investigated and discussed, extensively. We should also mention that the mixed Ising models of binary systems on the Bethe lattice consisting of spins $\sigma=\frac{1}{2}, S=1$; $\sigma=\frac{1}{2}, S=\frac{3}{2}$; $\sigma=\frac{3}{2}, S=\frac{5}{2}$; $\sigma=2, S=\frac{5}{2}$, etc. have been studied extensively in Refs. [36–40] and references therein.

The outline of the remainder of this paper is as follows: the description of the model and its formulation are given in Section 2. Namely, we give the exact expressions of the partition function, sublattice and total magnetizations, and magnetic susceptibilities. Thermal behavior of the sublattice and total magnetizations, and the phase diagrams are presented in Section 3. Finally, Section 4 is devoted to summary and conclusions.

## 2. Model formulation on the Bethe lattice

The mixed Ising model of the type ABC ternary system consisting of spins $\sigma=\frac{1}{2}$, $S=1$, and $m=\frac{3}{2}$ on the Bethe lattice is defined by the Hamiltonian

$$H=-J_1\sum_{\langle ij\rangle}\sigma_i S_j-J_2\sum_{\langle jk\rangle}S_j m_k-J_3\sum_{\langle ik\rangle}\sigma_i m_k+\Delta\left(\sum_j S_j^2+\sum_k m_k^2\right), \quad (1)$$

where each $\sigma_i$, $S_j$ and $m_k$ located at the sites $i$, $j$ and $k$ are a spin-$\frac{1}{2}$, with the discrete values $\pm\frac{1}{2}$, a spin-1, with the three discrete values $\pm1$ and 0, and a spin-$\frac{3}{2}$, with the four discrete values $\pm\frac{3}{2}$, and $\pm\frac{1}{2}$. $J_1$, $J_2$, and $J_3$ are the bilinear nearest-neighbor exchange interactions and we consider both ferromagnetic ($J_3>0$) and anti-ferromagnetic ($J_1<0$ and $J_2<0$) exchange interactions. $\Delta$ is the crystal-field interaction or single-ion anisotropy. The first three sums run over all nearest-neighbor (NN) pairs, the last sums run over all the spin-1 and spin-$\frac{3}{2}$ sites. In this ternary system case, we arrange the Bethe lattice such that the central spin is spin-$\frac{1}{2}$, $\sigma_0$; the second generation is spin-1, $S_0$; the third generation is a spin-$\frac{3}{2}$, $m_0$; the fourth generation is again spin-$\frac{1}{2}$, $\sigma_1$; the fifth generation is again spin-1, $S_1$; the sixth generation is again spin-$\frac{3}{2}$, $m_1$; so on to infinity, as seen in Fig. 1. We should also mention that the central spin has $q$ NN, i.e., the coordination number, which forms the second generation spins. Each spin in the second generation is joined to $(q-1)$ NN's. Therefore, in total the second generation has $q(q-1)$ NN spins that forms the third generation spins and so on to infinity. The Bethe lattice consideration for any model is based on the exact recursion relations.

The partition function is the main ingredient to obtain a formulation in terms of the recursion relations. Using the

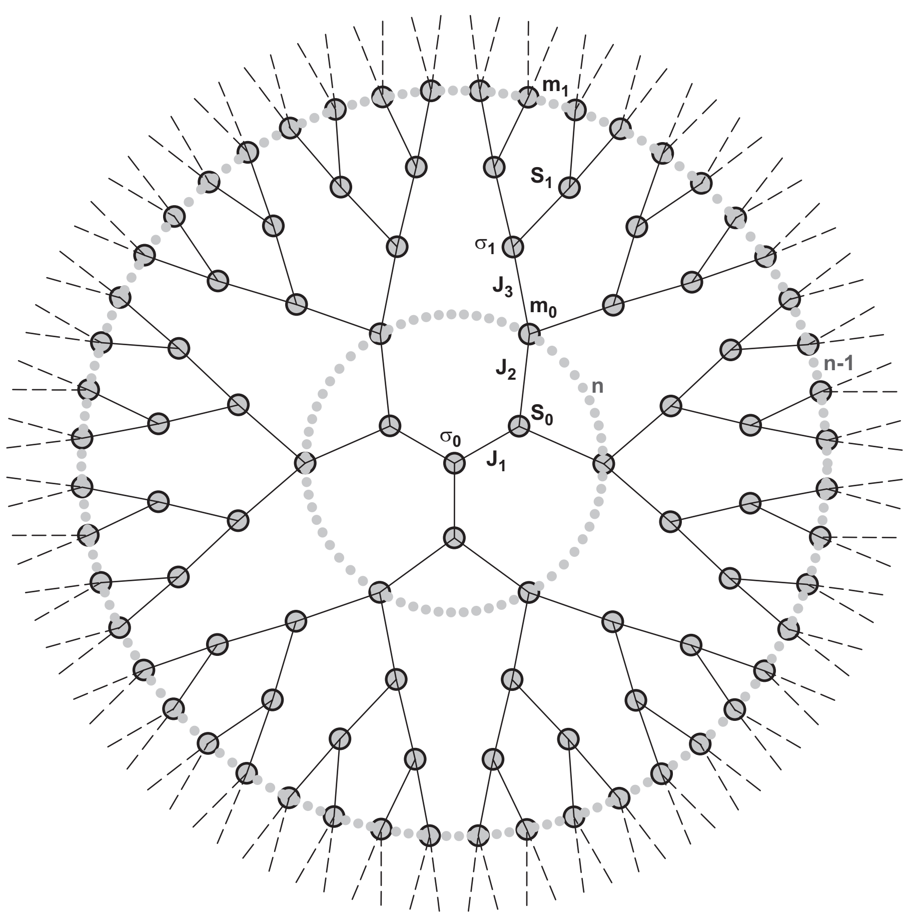


**Fig. 1.** Bethe lattice, or regular tree, of coordination number ($q$) 3 for the ternary system consisting of spins $\sigma=\frac{1}{2}$, $S=1$, and $m=\frac{3}{2}$. The Bethe lattice is arranged such that the central spin is spin-$\frac{1}{2}$, $\sigma_0$; the second generation is spin-1, $S_0$; the third generation is a spin-$\frac{3}{2}$, $m_0$; the fourth generation is again spin-$\frac{1}{2}$, $\sigma_1$; the fifth generation is again spin-1, $S_1$; the sixth generation is again spin-$\frac{3}{2}$, $m_1$; so on to infinity.