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# Magnetic field computation in a non-oriented sheet cross-section considering the hysteresis phenomenon

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ABSTRACT

The aim of the present work is the numerical computation of the average magnetic induction in the cross-section of a non-oriented 3% Si–Fe sheet by solving the magnetic diffusion equation. Jiles' dynamic model is used to describe the magnetization law. The obtained results are compared with those of the measurements carried out for frequencies of 0.5, 50, 200 and 500 Hz. A satisfactory agreement is obtained between both types of results.



## 1. Introduction

Electrical systems are basically made from ferromagnetic materials. These materials are exploited in the form of sheets to minimize the intensity of eddy currents. The most used alloy is mainly made up of iron and a small percentage of silicon that does not exceed 5% [1,2]. Several approaches have been suggested to study the magnetic phenomena in silicon–iron alloys. We mention for instance those of Bertotti [3], Bertotti and Mayergoyz [4], and Zirka et al. [5,6]. To optimize the sheet characteristics of silicon–iron alloys, such as reduction in magnetic losses and taking into account the skin effect, it is necessary to compute the magnetic field and the distribution of the magnetic flux density in the sheet cross-section in the dynamic mode. To do so, we chose to solve the diffusion equation resulting from the combination of Maxwell–Faraday and Maxwell–Ampere equations with Ohm's law equation [7–9]. To take into account the nonlinearity of the ferromagnetic material it is necessary to associate the diffusion equation with the magnetization law. The latter is represented by Jiles' dynamic model [10,11]. This model has the advantage of considering excess losses generated by the change in the configuration of magnetic domains during the magnetization process. The sheet under consideration is subjected to a sinusoidal surface field $H_s$. As modelling results we represent the hysteresis loops $B(H_s)$ for frequencies:0.5, 50, 200 and 500 Hz. The obtained results are compared with those determined by measurements [7,12]. The test bench used to obtain the measurements data is presented in Appendix A.

## 2. Magnetic field computation model

Let us suppose the sheet under consideration is placed in the $(oxyz)$ frame of reference. It is submitted to a magnetic field $H_s$ applied to its surface. The sheet cross-section is located in the $(oxy)$ plane (Fig. 1).

The magnetic field $H$ and the induction $B$ are perpendicular to the cross-section: $H=H_z(x,y,t)$ and $B=B_z(x,y,t)$. The corresponding two-dimensional diffusion equation is

$$\frac{\partial^2 H_z(x,y,t)}{\partial x^2}+\frac{\partial^2 H_z(x,y,t)}{\partial y^2}=\sigma\frac{\partial B_z(x,y,t)}{\partial t} \quad (1)$$

where $\sigma$ is the sheet's electrical conductivity. For its resolution, Eq. (1) simultaneously requires a discretization in time and space [13,14]. To do so, the finite elements method is used for the discretization in space and the Crank–Nicholson method for time discretization. The resulting set of matrix equations to be solved

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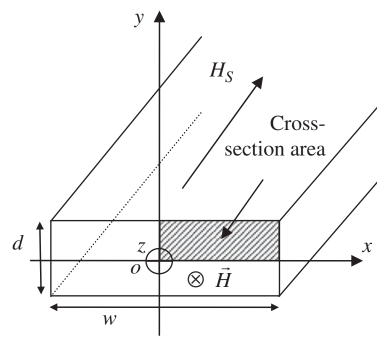


**Fig. 1.** Sheet geometry.

can be written as

$$(1-\beta)SH+T|_{t+(1-\beta)t}+\beta SH-T|_t-P=0 \tag{2}$$

where $0 \leq \beta \leq 1$.

In Crank–Nicholson's algorithm we take $\beta$=0.5.

$S$, $T$ and $P$ are matrices whose elements are

$$S^e_{ij}=\int_{\Omega^e}\vec{\nabla}N^e_i\vec{\nabla}N^e_j\,d\Omega \tag{3}$$

$$T^e_i=\sigma\frac{B_{t+(1-\beta)t}-B_t}{\Delta t}\int_{\Omega^e}N^e_i\,d\Omega \tag{4}$$

$$P^e_i=\int_{\Omega^e}N^e_i\frac{\partial H}{\partial n}\,d\Gamma \tag{5}$$

The indices $i$ and $j$ vary from 1 to $NN$, where $NN$ is the total number of nodes of the finite elements mesh. $H$ is an unknown vector giving the magnetic field values at the mesh nodes of the sheet cross-section. This vector is obtained by solving Eq. (2).

$N_i^e(x,y)$ are polynomial interpolation functions at the nodes of each element of the mesh with surface $\Omega^e$, $\Delta t$ is the time step, $n$ is the normal unit vector to the domain of study. To compute the distribution of magnetic field distribution at each time step, we used the Newton–Raphson algorithm. Therefore, Eq. (2) represents the residue vector $R$. Its Taylor series expansion near $H^{i-1}$, limited to the first order, gives

$$R(H^{i-1}+\Delta H^i)+\frac{\partial R}{\partial H}\Big|_{i-1}\Delta H^i=0 \tag{6}$$

where $H^{i-1}$ is the field vector value at iteration $i-1$. $\Delta H^i$ is the field vector variation at iteration $i$.

Eq. (6) can be written as follows:

$$-\frac{\partial R}{\partial H}\Big|_{i-1}\Delta H^i=R(H^{i-1}) \tag{7}$$

The resolution of this equation gives $\Delta H^i$. The field's vector value $H^i$, at iteration $i$, is obtained as

$$H^i=H^{i-1}+\Delta H^i \tag{8}$$

The derivative $\partial R/\partial H$ is expressed as follows:

$$\frac{\partial R}{\partial H}=(1-\beta)S+\frac{\partial T}{\partial H}\Big|_{t+(1-\beta)t}=(1-\beta)S+T' \tag{9}$$

where the elements of matrix $T'$ are given by

$$T'^e_{ij}=\frac{\sigma}{\Delta t}\frac{B_{t+(1-\beta)t}-B_t}{H_{t+(1-\beta)t}-H_t}\int_{\Omega^e}N^e_iN^e_j\,d\Omega \tag{10}$$

## 3. Magnetization law model *M*(*H*)

The magnetic hysteresis model chosen for this work is Jiles' dynamic model applied to conducting materials [10,11]. The model is expressed as a function of the differential susceptibility as follows:

$$\underbrace{\left(k\delta-\alpha\left(M_{an}-M+k\delta c\frac{dM_{an}}{dH_e}\right)\right)\left(\frac{dM}{dH}\right)}_{1}-\underbrace{\left(M_{an}-M+k\delta c\frac{dM_{an}}{dH_e}\right)}_{1}$$
$$+\underbrace{\left(\frac{\mu_0 d^2}{12\rho}\frac{dH}{dt}\right)\left(\frac{dM}{dH}\right)^2}_{2}+\underbrace{\left(\frac{\mu_0 GdwH_0}{\rho}\right)^{1/2}\left(\frac{dH}{dt}\right)^{1/2}\left(\frac{dM}{dH}\right)^{3/2}}_{3}=0 \tag{11}$$

To determine Jiles' parameters $k$, $\alpha$, $c$ and $a$ we used an iterative method as detailed in [15]. $\delta$ represents a factor that is equal to $\pm 1$. $\delta$=1 for $dH/dt>0$ and $\delta=-1$ for $dH/dt<0$. $M_{an}$ is the anhysteretic magnetization whose expression can be chosen in accordance with the model [12,16]. $\mu_0$ is the vacuum permeability. $\rho$ is the materials electrical resistivity. $d$ and $w$ are the thickness and width of the sheet, respectively; $G$=0.1356 [3]. $H_0$ is the magnetic field characteristic of the material microstructure and will be detailed in Section 4. Model (11) describes the dependence of the hysteresis on the frequency. It is composed of three terms: term 1 takes account of hysteresis losses, term 2 takes into account eddy current losses and term 3 considers excess losses. The last term is a consequence of wall movements between magnetic domains. It was treated in detail by Bertotti [17–19].

## 4. Determination of the parameter $H_0$

The parameter $H_0$ results from the theory of magnetic objects developed by Bertotti on fine grain materials [17–19]. In these materials, the coercive field fluctuations are controlled by grain size, each grain being considered as a magnetic object. Thus, we assume that the material cross-section is composed of $N_0$ magnetic objects and the distribution of local coercive fields of different magnetic objects is flat, and has a constant density value $1/H_0$. Hence, $H_0$ is the average minimum separation between different local coercive field values [18]. It is a field characteristic that determines the applied field ability to increase the number of effective active magnetic objects with an increase in magnetizing frequency [17]. It is very sensitive to the type of material. $H_0$ can easily be connected to the quasi-static macroscopic coercive field $H_c$, which is associated with the maximum magnetization $I_{max}(T)$ in the hysteresis loop [18,19]:

$$H_0=\frac{2s^2\langle I_S\rangle H_c}{SI_{max}} \tag{12}$$

$\langle I_S\rangle$ is the proper value of saturation magnetization. For a non-oriented grain sheet $\langle I_S\rangle$=1.7 T. $s$ represents the grain size and $S$ the sheet cross-section.

## 5. Results and discussion

The ferromagnetic sheet chosen to analyze the magnetic field whose model results from the combination of Eqs. (7), (8) and (11) is similar to the one given in [7,12]. The data in this application are as follows: $d$=0.5 mm, $w$=30 mm, $\rho=45\times10^{-8}\,\Omega$ m, $s$=60 μm, $I_{max}$=1.5 T, $H_0$=0.03, $a$=130.22 A/m, $k$=56.855 A/m, $\alpha=1.69\times10^{-04}$, $c=8.547\times10^{-03}$.

The finite elements analysis of the magnetic field in the sheet cross-section is carried out according to the flowchart presented in Fig. 2. Due to symmetry, only a quarter of the sheet cross-section needs to be analyzed (Figs. 1 and 3). The resulting mesh consists of 40 elements and 31 nodes. Dirichlet's non-homogeneous status is applied on the $\Gamma_1$ boundary (Fig. 3),