Journal of Magnetism and Magnetic Materials 320 (2008) 3196–3201

Contents lists available at ScienceDirect

Journal of Magnetism and Magnetic Materials

journal homepage: www.elsevier.com/locate/jmmm

Analytical characterization of detection SNR for PR target design in Viterbi-based receivers

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ARTICLE INFO

Available online 22 August 2008

PACS:
85.70.Kh
85.70.Li
43.38.Ne
43.38.Qg
84.40.Ua

Keywords:
Generalized partial-response (GPR) target
Bit-error-rate (BER)
Detection SNR
Perpendicular recording

ABSTRACT

In this paper, we address the problem of designing the optimum partial-response (PR) target that results in the best bit-error-rate (BER) performance in Viterbi-like detectors when applied for signal detection in digital magnetic recording systems. We present a detailed analysis of the effective detection signal-to-noise ratio (SNR_{eff}), which is closely related to the BER performance of a Viterbi detector. We show that SNR_{eff} is a concave function with a unique global maximum corresponding to the magnitude frequency response of the optimum targets. Thus, any simple search approach is guaranteed to reach the optimum target. We consider the cases of single dominant error event and multiple dominant error events. Numerical and simulation results are presented to corroborate our analytical results for perpendicular magnetic recording channels.

1. Introduction

The most widely used signal detection technique in current high-performance digital magnetic recording systems is ‘partial-response maximum-likelihood’ (PRML) and its derivatives [1,2]. The PRML approach employs a linear PR equalizer to shape the natural channel into a short-length PR target. A Viterbi detector (VD) or one of its iterative versions, which is tuned to the PR target, works on the equalizer output to detect the stored bit sequence.

When the noise at the VD input is additive white Gaussian, its detection performance is optimum in the sense of the maximum likelihood [3]. However, the use of equalizer and the presence of media noise result in non-white noise at VD input, thus making the VD sub-optimum. To achieve good detection performance, the PR target should be well designed to minimize mis-equalization and noise correlation. Conventional standard targets with integer-valued coefficients, such as the well-known Class I, Class II and Class IV PR targets [1,4], are selected by inspecting the match between the target and the natural channel. Compared to these, the generalized PR (GPR) targets with real-valued coefficients can provide better match to the natural channel, and significantly reduce noise enhancement. Different approaches have been proposed to design GPR targets. Among these, the most widely used method is joint optimization of target and equalizer based on the minimum mean square error (MMSE) criterion [5,6], i.e. by minimizing the total noise power at VD input. Another popular approach is noise prediction [2] wherein the overall target is the cascade of a short primary target and a noise-whitening filter (or a bank of filters [7]) that reduces the noise correlation at VD input. GPR targets that suppress media noise are reported in Ref. [8].

To evaluate the detection performance for a given target, we may conduct bit-error-rate (BER) simulations or compute the corresponding effective detection signal-to-noise ratio, SNR_{eff}. It has been shown using theory and simulations that the SNR_{eff} is equivalent to the BER performance of VD [6,9]. Therefore, the target that maximizes SNR_{eff} should result in the best BER. The SNR_{eff} criterion, which takes into account the noise correlation at VD input, is strongly dependent on the error event distribution at VD output. Target design based on SNR_{eff} has been investigated in Ref. [6] by search approach and in Refs. [10,11] by analytical approach. Search approaches for determining the global optimizer of SNR_{eff} are computationally very expensive [6]. A partial analytical solution for the case of single-bit error event was reported in Ref. [10]. In Ref. [11], the authors reported a complete analytical solution for a given dominant error event (single or multiple errors).

The SNR_{eff} is a very complicated cost function and no study has been reported on the characterization of its stationary points. Further, the dominant error event may change with choice of the

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doi:10.1016/j.jmmm.2008.08.091

target and a group of error events may become dominant rather than a single event. Therefore, it is necessary to investigate the shape of the surface of $\mathrm{SNR_{eff}}$ and characterize the nature of its stationary points, so as to develop meaningful numerical or analytical approaches to design optimum PR targets. In this paper, we present analytical characterization of the stationary points of the surface defined by $\mathrm{SNR_{eff}}$. We show that all the targets that maximize $\mathrm{SNR_{eff}}$ are global optimizers of $\mathrm{SNR_{eff}}$ and they have the same magnitude frequency response. Our analysis provides helpful guidelines for the design of optimum PR targets.

The paper is organized as follows. Section 2 describes the channel model used here and introduces the cost function $\mathrm{SNR_{eff}}$. Section 3 provides characterization of the surface of $\mathrm{SNR_{eff}}$ for a given dominant error event. Section 4 generalizes this by accounting for the possibility of multiple dominant error events and the dependence of dominant error events on the target. Section 5 presents some numerical and simulation results to corroborate our analytical results. Section 6 concludes the paper.

2. System model and effective detection SNR

A block diagram of the system model is shown in Fig. 1. The signal at the equalizer input is given by

$$z_k = \sum_i h_i a_{k-i} + v_k + m_k, \tag{1}$$

$$m_k = \sum_i (a_{k-i} - a_{k-i-1})\tau_{k-i} f_i, \tag{2}$$

where the binary input data $a_k \in \{-1,+1\}$ is random with zero mean, v_k is electronics noise modeled as white Gaussian with variance σ_v^2, m_k is media noise modeled as first-order position-jitter noise, τ_k is position-jitter associated with the transition $a_k - a_{k-1}$, and h_i and f_i are sampled bit response and step response, respectively, of the channel. The channel response is modeled as

$$f(t) = \frac{2V_\mathrm{p}}{\pi}\arctan\left(\frac{2t}{T_{50}}\right), \tag{3}$$

$$f_i = f(iT), h_i = f(iT + 0.5T) - f(iT - 0.5T), \tag{4}$$

where V_p is the saturation amplitude, T_{50} is the time required for $f(t)$ to rise from $-0.5V_\mathrm{p}$ to $+0.5V_\mathrm{p}$, and T is one bit period. The position jitter, τ_k, is modeled as white Gaussian with variance σ_τ^2, and we truncate the distribution such that $|\tau_k| < 0.5T$. In this paper, jitter noise is specified in terms of percentage jitter, i.e. $(100\sigma_\tau/T)\%$. We define the channel SNR based on electronics noise only as $10\log_{10}(V_\mathrm{p}^2/\sigma_v^2)$.

The equalizer, w_i, shapes the channel bit response, h_i, into the PR target, g_i, and k_0 accounts for the delay from channel input to equalizer output. The error, e_k, is the total noise at VD input and it includes mis-equalization and channel noises, m_k and v_k, filtered by the equalizer.

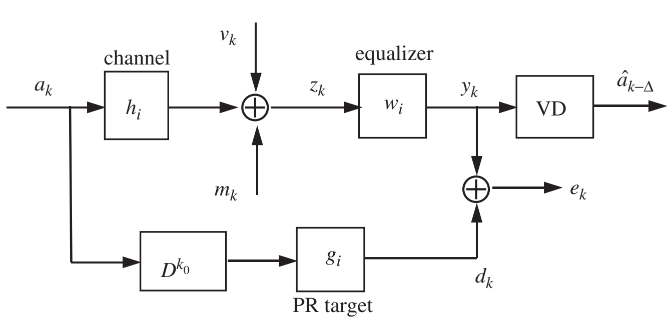

Fig. 1. System model of magnetic recording channel with PRML receiver.

At medium-to-high channel SNRs, the BER of VD can be accurately predicted by [6,12]

$$P_\mathrm{b} = \sum_{\{\text{all possible } \underline{\varepsilon}\}} Q\left(\frac{\underline{\tilde{\varepsilon}}^\mathrm{T}\underline{\tilde{\varepsilon}}}{2\sqrt{\underline{\tilde{\varepsilon}}^\mathrm{T} R \underline{\tilde{\varepsilon}}}}\right) w_\mathrm{H}(\underline{\varepsilon}) P_\mathrm{r}(\underline{\varepsilon}), \tag{5}$$

where superscript 'T' denotes matrix transpose, $Q(\cdot)$ is the tail integral of the Gaussian probability density, R is the autocorrelation matrix of total noise at VD input, the column vector $\underline{\tilde{\varepsilon}}$ is an error event sequence $\underline{\varepsilon}$ filtered by the PR target, $w_\mathrm{H}(\underline{\varepsilon})$ is the Hamming weight of $\underline{\varepsilon}$, and $P_\mathrm{r}(\underline{\varepsilon})$ is the probability of data patterns supporting the error event $\underline{\varepsilon}$. As $Q(x)$ exponentially decreases with increasing x, we can approximate the BER as

$$P_\mathrm{b} \approx Q\left(\frac{\underline{\tilde{\varepsilon}}_\mathrm{d}^\mathrm{T}\underline{\tilde{\varepsilon}}_\mathrm{d}}{2\sqrt{\underline{\tilde{\varepsilon}}_\mathrm{d}^\mathrm{T} R \underline{\tilde{\varepsilon}}_\mathrm{d}}}\right) w_\mathrm{H}(\underline{\varepsilon}_\mathrm{d}) P_\mathrm{r}(\underline{\varepsilon}_\mathrm{d}), \tag{6}$$

where $\underline{\tilde{\varepsilon}}_\mathrm{d}$ is the dominant error event that minimizes the argument of $Q(\cdot)$ in Eq. (5). Based on Eq. (6), the effective detection SNR, defined as

$$\mathrm{SNR_{eff}} = \left(\frac{\underline{\tilde{\varepsilon}}_\mathrm{d}^\mathrm{T}\underline{\tilde{\varepsilon}}_\mathrm{d}}{2\sqrt{\underline{\tilde{\varepsilon}}_\mathrm{d}^\mathrm{T} R \underline{\tilde{\varepsilon}}_\mathrm{d}}}\right)^2 = \frac{1}{4}\frac{(\underline{\tilde{\varepsilon}}_\mathrm{d}^\mathrm{T}\underline{\tilde{\varepsilon}}_\mathrm{d})^2}{\underline{\tilde{\varepsilon}}_\mathrm{d}^\mathrm{T} R \underline{\tilde{\varepsilon}}_\mathrm{d}} \tag{7}$$

arises as an equivalent measure of the BER performance of VD. In other words, if the PR target and equalizer are chosen to maximize $\mathrm{SNR_{eff}}$, the resulting PRML detection system is expected to achieve the lowest BER.

3. Global optimality of the target solution

In this section, we present an analysis to show that a target (and the corresponding infinite-length optimum equalizer) that maximizes $\mathrm{SNR_{eff}}$ is a globally optimum target and all such optimum targets have the same magnitude frequency response.

Using frequency domain, we can express $\mathrm{SNR_{eff}}$ as

$$\mathrm{SNR_{eff}} = \frac{0.25\left(\int_{-0.5}^{0.5}|GE_\mathrm{d}|^2\,\mathrm{d}\Omega\right)^2}{\int_{-0.5}^{0.5}|GE_\mathrm{d}|^2(P_\mathrm{n}|W|^2 + P_\mathrm{a}|WH - Ge^{-\mathrm{j}2\pi\Omega k_0}|^2)\,\mathrm{d}\Omega}, \tag{8}$$

where G, W, H and E_d are discrete-time Fourier transforms of PR target g_i, equalizer w_i, channel bit response h_i, and dominant error event $\underline{\tilde{\varepsilon}}_\mathrm{d}$, respectively, Ω is the frequency normalized by bit rate $1/T$, and P_a and P_n are power spectral densities of input data a_k and total noise at equalizer input, respectively. (For notational brevity, we omit the argument '$e^{\mathrm{j}2\pi\Omega}$' from the frequency-domain quantities in Eq. (8) and hereafter.)

For a given dominant error event, only the denominator of Eq. (8) depends on the equalizer, W. Since the denominator of Eq. (8) is quadratic in W, using calculus of variations [12], the optimum equalizer (infinite length) that maximizes $\mathrm{SNR_{eff}}$ can be obtained as

$$W_0 = \frac{P_\mathrm{a}H^* e^{-\mathrm{j}2\pi\Omega k_0}}{P_\mathrm{n} + P_\mathrm{a}|H|^2}G, \tag{9}$$

where superscript '*' denotes complex conjugation. Thus, the optimum equalizer based on $\mathrm{SNR_{eff}}$ is same as the MMSE equalizer [12]. Using Eq. (9) in Eq. (8) yields

$$\mathrm{SNR_{eff}} = \frac{\left(\int_{-0.5}^{0.5}|G|^2|E_\mathrm{d}|^2\,\mathrm{d}\Omega\right)^2}{\int_{-0.5}^{0.5}(|G|^4 K)\,\mathrm{d}\Omega}, \quad K = \frac{4|E_\mathrm{d}|^2 \mathrm{P_a P_n}}{\mathrm{P_n} + \mathrm{P_a}|H|^2}. \tag{10}$$