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Extraction of hysteresis loops from main magnetization curves

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Abstract

In firm catalogues,the basic properties of magnetic materials are often described by their main magnetization curves. Such curves may be used for the first analysis of circuits containing ferromagnetic cores. The analysis will be more accurate, if the curves are transformed into the families of hysteresis loops. To enable the reconstruction of such loops, we formulate a simple model of hysteresis, making main magnetization curve directly dependent on coercivity. In this way we can approximate hysteresis loops of most typical materials in a pretty wide range of magnetization. Application of variable coercivity enables extension of the model to stronger fields.

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1. Introduction

Manufacturers of magnetic materials usually characterize their products by the main magnetization curves, joining the tips of symmetrical quasi-static hysteresis loops. More accurate analysis of circuits containing ferromagnetic materials requires transformation of the curves into the families of hysteresis loops. In general, this is not possible, because the same tips correspond to different loops. If, however, the considered loops have the same typical shape, then the loops stretched between the same extreme points differ mainly in their width. In this case, the transformation may be done with the use of hysteresis model leading to an analytical approximation of main magnetization curve dependent directly on coercivity.

Due to the variety of materials, it is difficult to find a universal analytical formula determining the curves with satisfactory accuracy. Purely mathematical methods use for instance piecewise linear construction [1], Frölich curve [2], rational fractures [3], inverse tangent functions [4], Fourier and exponential series [5,6]. Some other approximations result from the general descriptions of hysteresis, such as the Preisach [7,8], Jiles-Atherton [9] or Takács [10] models. Merely the last model gives the direct analytical expression for the main magnetization curve. Unfortunately, for strong fields it leads to the excessive rise of magnetization due to the linear component of the model.

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2. Mathematical description of magnetization

Let us assume that magnetization consists of two independent superimposed saturation processes, in which the continuous and slow, reversible process is in a certain region screened by the more violent, irreversible process. Initial stages of magnetization are well described by the empirical Rayleigh relation [11,12], which may be written in equivalent forms

$$M = \alpha h + \beta |h| h = M_a \frac{h}{3a} + M_b \frac{|h|}{b} \frac{h}{3b}, \tag{1}$$

where h is field intensity. Assuming that in further stages both processes gradually slow down and approach saturation, we replace the fractions in Eq. (1) by the classical Langevin function [13]

$$L(x) = \coth(x) - \frac{1}{x}. \tag{2}$$

At the same time the absolute value of the field in the second term will be replaced by the field amplitude H

corresponding to the considered hysteresis loop. In accordance with this, the anhysteretic magnetization is approximated by the relation

$$M(h,H) = M_a L\left(\frac{h}{a}\right) + M_b L\left(\frac{3H}{b}\right) L\left(\frac{h}{b}\right), \tag{3}$$

where M_a, M_b denote the two components of saturation magnetization

$$M_s = M_a + M_b \tag{4}$$

and parameters a, b determine the rates of the approach to saturation.

In order to get hysteresis loops, the irreversible component of Eq. (3) is translated horizontally and then vertically in the opposite directions, like in the Takács model [10]. Assuming that the horizontal displacement is equal to the coercivity c, the new curves satisfy the equation

$$M_\pm(h,H) = M_a L\left(\frac{h}{a}\right) + M_b L\left(\frac{3H}{b}\right) L\left(\frac{h \pm c}{b}\right) \pm d. \tag{5}$$

In the next step, the vertical shift d is determined from the condition that the two branches of the loop coincide at the point corresponding to the field amplitude H

$$M_+(H,H) = M_-(H,H). \tag{6}$$

Hence, the shift becomes the function of field amplitude

$$d(H) = \frac{M_b}{2} L\left(\frac{3H}{b}\right)\left[L\left(\frac{H-c}{b}\right) - L\left(\frac{H+c}{b}\right)\right] \tag{7}$$

and Eq. (5) may be written in the final form

$$M_\pm(h,H) = M_a L\left(\frac{h}{a}\right) + M_b L\left(\frac{3H}{b}\right) L\left(\frac{h \pm c}{b}\right) \pm d(H). \tag{8}$$

At the tips of loops ($h = H$) the two branches reduce to the main magnetization curve

$$M(H) = M_a L\left(\frac{H}{a}\right) + \frac{M_b}{2} L\left(\frac{3H}{b}\right) \times \left[L\left(\frac{H+c}{b}\right) + L\left(\frac{H-c}{b}\right)\right] \tag{9}$$

or the corresponding flux density curve

$$B(H) = \mu_0[H + M(H)]. \tag{10}$$

The area inside the loop (8) determines energy loss in unit volume of a core per one cycle of magnetization

$$W(H) = \mu_0 \int_{-H}^{H} [M_+(h,H) - M_-(h,H)]\,\mathrm{d}h. \tag{11}$$

3. Approximation of typical hysteresis loops

Model parameters M_a, M_b, a, b, c will be determined from the condition

$$\sum [B(H_i) - B_i]^2 = \text{minimum}, \tag{12}$$

where $B(H_i)$ are values computed from Eqs. (9), (10), and B_i—experimental flux densities related to field amplitudes H_i. Additionally we demand that all computed loops (8) do not intersect one another

$$M_-(H_i, H_{i+1}) < M_-(H_i, H_i). \tag{13}$$

3.1. Silicon steel

The experimental dependence of flux density on field amplitude for transformer steel with 0.2% silicon content is presented by the data in Table 1, read from the experimental magnetization curve [14]. Using (12) and (13) we get

$$M_a = 0.537\,\text{MA/m}, \quad M_b = 1.163\,\text{MA/m},$$
$$a = 5025\,\text{A/m}, \quad b = 27.6\,\text{A/m}, \quad c = 127.7\,\text{A/m}$$

and consequently main magnetization curve, hysteresis loops and related energy losses presented in Figs. 1–3.

In the same way we extract model parameters

$$M_a = 0.475\,\text{MA/m}, \quad M_b = 1.075\,\text{MA/m},$$
$$a = 4830\,\text{A/m}, \quad b = 16.7\,\text{A/m}, \quad c = 67.4\,\text{A/m}$$

from the magnetization curve of steel having about 4% of silicon content [14]. Much lower value of coercivity reveals

Table 1
Successive points of the magnetization curve of steel with 0.2% silicon content

H (A/m)	0	40	80	110	140	200	300
B (T)	0	0.06	0.21	0.50	0.75	1.04	1.23
H (A/m)	400	800	2000	10,000	20,000	40,000	60,000
B (T)	1.34	1.46	1.59	1.79	1.92	2.08	2.15

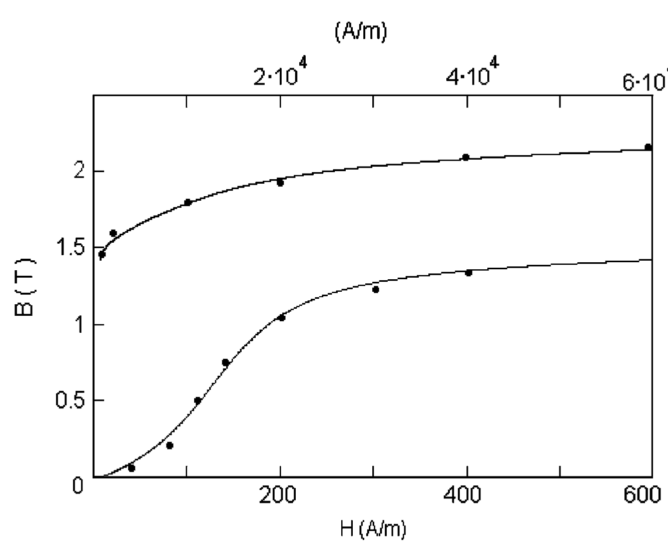

Fig. 1. Magnetization curve (9) of steel with 0.2% of silicon content (points–experimental data).