Journal of Magnetism and Magnetic Materials 321 (2009) 3007–3013

Contents lists available at ScienceDirect

Journal of Magnetism and Magnetic Materials

journal homepage: www.elsevier.com/locate/jmmm

A renormalization-group analysis of a spin-1 Ising ferromagnet with competing crystal-field and repulsive biquadratic interactions

Daniel P. Snowman *

Department of Physical Sciences, Rhode Island College, Providence, RI 02908, USA

ARTICLE INFO

Article history:
Received 18 November 2008
Received in revised form
15 April 2009
Available online 3 May 2009

PACS:
5.70.Fh
64.60.−i
K75.10.Nr
5.50.+q

Keywords:
Frustration
Renormalization
Blume-Emery-Griffiths

ABSTRACT

Phase diagrams have been produced and critical exponents calculated for a Blume–Emery–Griffiths system with competing biquadratic and crystal-field interactions with uniform ferromagnetic bilinear interactions. This competition directly effects the clustering and density of nonmagnetic impurities. These results have been produced using renormalization-group methods with a hierarchical lattice. A series of planes of constant, repulsive biquadratic coupling have been probed while varying the temperature and concentration of annealed vacancies in the system. The sinks have been analyzed and interpreted, and critical exponents calculated for the higher order transitions.

1. Introduction

The Blume–Emery–Griffiths (BEG) model [1] is a spin-1 Ising model with bilinear (J_{ij}), biquadratic (K_{ij}) and crystal-field interaction (Δ_{ij}) terms as shown in the Hamiltonian

$$-\beta H = \sum_{\langle ij\rangle} J_{ij} s_i s_j + \sum_{\langle ij\rangle} K_{ij} s_i^2 s_j^2 - \sum_{\langle ij\rangle} \Delta_{ij}(s_i^2 + s_j^2) \quad \text{with } s_i = 0, \pm 1 \tag{1}$$

The bilinear interactions directly effect magnetic ordering, whereas the other two terms directly effect the relative density ($\sim\Delta/J$) and clustering ($\sim K/J$) of occupied sites in the system. The model works well for systems characterized with fluctuations in both density and magnetization

$$\sum_{\langle ij\rangle} H_{ij}(s_i + s_j) + \sum_{\langle ij\rangle} L_{ij}(s_i^2 s_j + s_i s_j^2) \tag{2}$$

In addition to the bilinear, biquadratic and crystal-field interaction terms we must also consider odd sector contributions to the Hamiltonian, as shown in Eq. (2). These contributions must be included in order to obtain a complete description of higher order phase transitions arising in our system.

Each contribution to the Hamiltonian involves a summation over nearest-neighbor $\langle ij\rangle$ pairs of our lattice unit structure including the magnetic (H) and crystal-field (Δ) interactions. Normally, these field interactions (H and Δ) involve a summation over lattice sites; however, this investigation has changed the summation from sites to bonds for computational convenience. The net result of this shift being Δ in Eq. (1), and H in Eq. (2), represents the chemical potential, and magnetic field, per bond divided by two.

The BEG model has been extended beyond its original application, to the superfluid transition in He^3-He^4 mixtures [1], to consider several other systems. In particular, structural glasses [2], microemulsions [3], binary fluids, materials with mobile defects, semiconductor alloys [4], frustrated percolation [5], and aerogels are but a few systems that have been better understood by employing the BEG model.

The criticality and phase diagrams can become quite complex due to underlying competing interactions in various Blume–Emery–Griffiths systems. Many different types of competing interactions have been the focus of previous studies using the Blume–Emery–Griffiths model in conjunction with mean-field methods [6–9] and/or renormalization-group techniques [10–16].

Renormalization-group techniques in conjunction with hierarchical lattices have been used to probe the effects of competing bilinear interactions [11] in a spin-$\frac{1}{2}$ Ising model, competing biquadratic interactions in a dilute Ising ferromagnet [14], competing bilinear interactions in a BEG system [13], and

* Tel.: +1 401 456 9710; fax: +1 401 456 8379.
E-mail address: dsnowman@ric.edu

0304-8853/$ - see front matter © 2009 Elsevier B.V. All rights reserved.
doi:10.1016/j.jmmm.2009.04.056

simultaneous competition between crystal-field and biquadratic interactions in a BEG ferromagnet with attractive biquadratic interactions [12]. Each of these studies included tuning parameters allowing for the degree of frustration to be decreased, increased or maximized.

Previous studies have considered the effects of random crystal fields using real-space renormalization-group methods [15,16] and mean-field approximations [15,17] for both Blume–Capel and Blume–Emery–Griffiths model Hamiltonians, respectively. Still, other studies have considered the effects of quenched random bonds [18] and quenched random fields [19] upon the criticality and phase diagrams in BEG systems.

The present study complements these earlier investigations as it considers a Blume–Emery–Griffiths system with competing biquadratic ($\pm K$) and crystal-field ($\pm\Delta$) interactions with uniform ferromagnetic bilinear interactions (J). Phase diagrams have been produced, and critical exponents calculated, for a series of planes of constant, repulsive biquadratic coupling while varying the temperature and concentration of annealed vacancies in the system. These results have been produced using renormalization-group methods with a hierarchical lattice.

2. Hierarchical lattices and renormalization-group theory

The construction of a general hierarchical lattice is depicted in Fig. 1. Fig. 2 illustrates the construction of the hierarchical lattice [20,21] used for this study. The general process, for constructing an infinite hierarchical lattice, consists of replacing each nearest-neighbor interaction in the basic unit, by the basic unit itself.

Since hierarchical lattices produce exact renormalization-group recursion relations, phase diagrams can be produced and critical exponents determined very precisely. Thus, the results presented in this paper are exact on the rather specialized hierarchical lattice. Or, the calculations of phase diagrams and critical exponents may be considered as approximations on more realistic lattices. A wide range of complex problems have been probed and further understood using hierarchical lattices. For instance, spin glass [13,22], frustrated [11,12,14], random-bond [23], directed-path [24], random-field [25] and dynamic scaling [26] systems have all been subjected to study using hierarchical lattices.

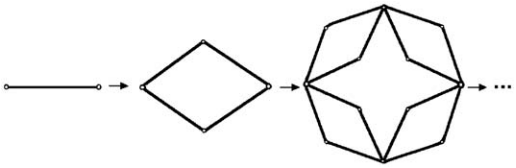

Fig. 1. An infinite hierarchical lattice is generated from a basic unit by repeatedly replacing each nearest-neighbor interaction by the basic unit itself [20].

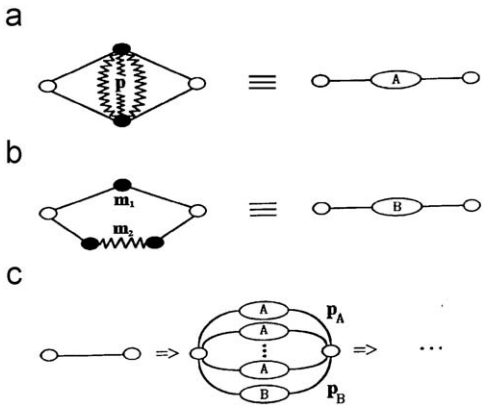

Fig. 2. Construction of the hierarchical lattice. Solid lines represent (J,K,Δ) nearest-neighbor site interactions, whereas jagged lines represent $(J,-K,-\Delta)$ nearest-neighbor site interactions. The general process, for constructing an infinite hierarchical lattice, consists of replacing each nearest-neighbor interaction in the basic unit, by the basic unit itself. (Reprinted from Journal of Magnetism and Magnetic Materials, 314, 69–74 (2007), D.P. Snowman, with permission from Elsevier.)

Reversing the construction process, shown in Figs. 1 and 2, yields renormalization-group relationships for the various coupling coefficients as internal degrees of freedom are eliminated with each renormalization. The internal degrees of freedom (i.e. internal spin sites) are represented by solid black dots in Fig. 2a and b, and, by σ_i, σ_j in Eq. (5).

Recursion relations are derived by demanding that the partition function be preserved at each length scale. The new effective interactions J', K', and Δ' are separated by a distance l', which is b lattice constants in the original system—where b is the length rescaling factor of the renormalization-group transformation

$$\zeta_{l'}(J',K',\Delta') = \zeta_l(J,K,\Delta) \quad \text{with } l' = bl \tag{3}$$

$$\zeta_l = \sum_{s_i,s_j} \exp[-\beta H] = \sum_{s_i,s_j} R_l(s_i,s_j) \tag{4}$$

$$\text{with } R_l(s_i,s_j) = \sum_{\sigma_i,\sigma_j} \exp[-\beta H] \tag{5}$$

$$\zeta_{l'} = \sum_{s'_i,s'_j} \exp[-\beta H'] = \sum_{s'_i,s'_j} R_l(s'_i,s'_j) \tag{6}$$

$$\text{with } R_{l'}(s'_i,s'_j) = \sum_{s'_i,s'_j} \exp[J' s_i s_j + K' s_i^2 s_j^2 - \Delta'(s_i^2+s_j^2) + \widetilde{G}'] \tag{7}$$

where $\widetilde{G}'$ is a constant used to calculate the free energy.

The contributions, $R_l(s_i,s_j)$ and $R_{l'}(s'_i,s'_j)$, to the two partition functions ($\zeta_{l'}$ and ζ_l), at length scales l and l', correspond to the same fixed configuration of end spins, s_i,s_j and s'_i,s'_j. Equating these contributions yields renormalization-group relationships relating the coupling coefficients at the two length scales, l and l': $J'(J,K,\Delta), K'(J,K,\Delta)$, and $\Delta'(J,K,\Delta)$. The reader is directed to Section 4 for a derivation of these relations.

These recursion relations are used to map phase diagrams and probe the nature of transitions by renormalizing the system from a set of initial values of J,K and Δ, until a sink for the renormalization-group trajectory is reached

$$J' = R_J(J,K,\Delta) \tag{8}$$

$$K' = R_K(J,K,\Delta) \tag{9}$$

$$\Delta' = R_\Delta(J,K,\Delta) \tag{10}$$

Table 1
Phases and corresponding sinks.

Phase	Sink	Characteristics
Paramagnetic I	$J \to 0$ $K \to 0$ $\Delta \to -\infty$	Mag. disorder Dense sublattice I Dilute sublattice II
Ferromagnetic I	$J \to +\infty$ $K \to -\infty$ $\Delta \to -\infty$	Mag. order Dense sublattice I $\lvert\Delta\rvert \gg \lvert K\rvert = \lvert J\rvert$
Paramagnetic II	$J \to 0$ $K \to 0$ $\Delta \to +\infty$	Mag. disorder Dilute sublattice I Dense sublattice II
Ferromagnetic II	$J \to +\infty$ $K \to -\infty$ $\Delta \to -\infty$	Mag. order Dense sublattice II $\lvert\Delta\rvert \gg \lvert K\rvert = \lvert J\rvert$