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An experimental study of the properties of magnetoinductive waves in the presence of retardation

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Abstract

Magnetoinductive (MI) waves owe their existence to the magnetic coupling between metamaterial elements. First experiments confirming the existence of MI waves were carried out on capacitively loaded loops and Swiss Rolls about three orders of magnitude smaller than the operating wavelengths (5–15 m) so that the radiation effects did not play any significant role. In the present paper MI waves are studied experimentally on various types of split ring resonators of about 1 cm diameter operating in the microwave region between 1 and 2 GHz. Our results prove that retardation has a significant effect upon the propagation of MI waves.

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1. Introduction

Waves propagating on coupled LC circuits were investigated by Atabekov [1] and Silin and Sazonov [2] in the 1960s as simple examples of periodic filters and slow wave structures, respectively. The properties of these waves were studied in more detail [3–6] four decades later as a by-product of the research on metamaterials. Due to the coupling between the elements they were called magnetoinductive (MI) waves. Applications as waveguides [7,8], delay lines [9], phase shifters [10] and lenses [11] have already been proposed. Experiments confirming the existence of MI waves were conducted in the MHz region on arrays consisting of capacitively coupled loops and of 'Swiss Rolls' [5,6].

The aim of the present paper is to extend the detailed experimental studies to a higher frequency region at which the size of the structure can no longer be regarded as small relative to the electromagnetic wavelength. The elements investigated belong to the broader family of split ring resonators (SRRs), introduced by Hardy and Whitehead [12]. They were made popular by Pendry et al. [13] who proved that they can, in a certain frequency range, offer negative effective permeability.

2. The elements

We have looked at the properties of two types of elements. SRRs on a dielectric substrate as proposed by Pendry et al. [13], and a more recent realization [14] using short metallic pipes of 5 mm height. The dimensions and resonant frequencies of the five elements investigated are shown in Table 1. They will be referred to as A_d, B_d, A_p, B_p and C_p. The subscripts d and p stand for dielectric and pipe samples, respectively. Note that A_d and A_p are doubly split double rings, B_d and B_p are singly split double rings with the outer ring open and C_p is a split single ring. They can be

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Table 1
The metamaterial elements investigated, their dimensions in mm and resonant frequencies in GHz

	A_d	B_d	A_p	B_p	C_p
					
r_i	6.0	6.0	7.1	7.1	—
r_e	9.7	9.7	10.5	10.5	10.5
w_i	2.0	2.0	0.8	0.8	—
w_e	2.0	2.0	1.0	1.0	1.0
g_i	3.0	—	1.0	—	—
g_e	3.0	3.0	1.0	1.0	1.0
$\omega_0/2\pi$	1.56	1.86	1.44	1.87	1.76

described by the parameters of r_i, and r_e, mean radii of inner and outer ring, w_i and w_e, wall thicknesses of the inner and outer ring, g_i and g_e, gap widths of the inner and outer ring. The dielectric constant of the substrate is $\varepsilon_r = 2.4$.

3. Measurements

The experiments were carried out in the frequency range of 300 kHz to 6 GHz on linear arrays consisting of 2, 8 and 16 elements. The first element in the array was excited by a coil protruding from the end of a coaxial cable placed parallel with and directly below the element (transmitter). The amplitude and phase of the magnetic field along the array was then measured by a similarly constructed probe (receiver) moving along the axis of the array at a certain distance above the elements as shown schematically in Fig. 1. Both transmitter and receiver coils were connected to a network analyzer taking measurements at 401 frequency points.

4. Results for two elements

Taking two identical elements at a distance of d from each other we measured the high-frequency magnetic fields above each of the elements. Assuming that the fields are proportional to the currents, we may derive from that measurement, I_2/I_1, the ratio of the two currents which from a simple theory is of the form

$$I_2/I_1 = (2M/L)(1 - \omega_0^2/\omega^2 + j/Q)^{-1}, \quad (1)$$

where ω is the frequency, $\omega_0 = (LC)^{-1/2}$ is the resonant frequency, $Q = \omega L/R$ is the quality factor, L, C and R are the inductance, capacitance and resistance of an element and M is the mutual inductance between the two elements. The value of Q may be obtained measuring the resonance curve of a single element which may be expected to be independent of the distance between the elements. Hence Eq. (1) determines effectively the variation of the mutual inductance against distance. In the presence of retardation the mutual inductance is complex and so is the ratio I_2/I_1.

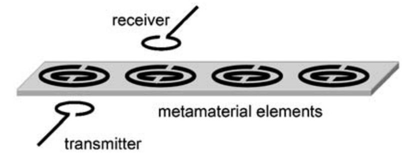

Fig. 1. Experimental setup.

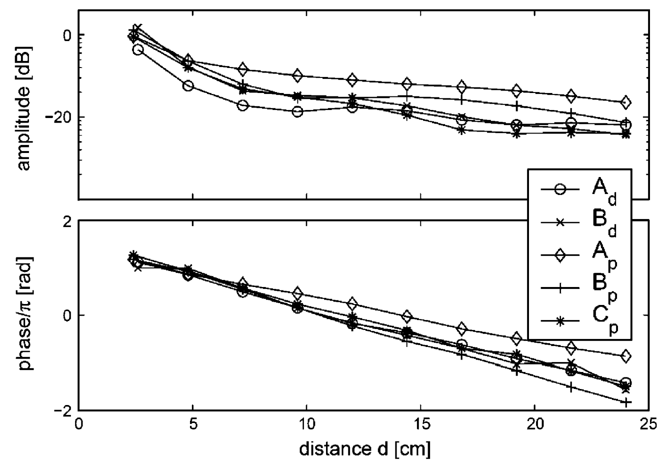

Fig. 2. Two-element measurements. The ratio I_2/I_1 (amplitude and phase) at ω_0 versus distance.

For the five cases mentioned the modulus and the phase angle of the current ratio at the resonant frequency are plotted in Fig. 2 as a function of distance.

As may be expected the modulus of M declines with distance. The longest distance between the elements may be seen to be 24 cm and the highest resonant frequency considered is 1.87 GHz, hence in this case the total length is about one and a half wavelength. Clearly we are well within the region where retardation needs to be taken into account. In a future paper we shall have detailed comparison between experimental and theoretical results. For the moment we shall give only one comparison which is shown in Fig. 3 where besides the experimental results for the A_p element the theoretical value of I_2/I_1 at resonance is also plotted with and without retardation. The theoretical values were obtained by the following algorithm. Assuming the elements in form of filament currents of radius $r_0 = 1$ cm (with the resonant frequency of 1.4 GHz and the quality factor of 50 from single element measurements) the mutual inductance between the elements was obtained by numerical integration of the expression for the vector potential [15]. This involves two consecutive line integrals which, if retardation is neglected, can be shown to reduce to a single integration of an expression in terms of elliptic integrals [4]. All numerical integrations were performed using Matlab. It may be clearly seen that retardation must be included even for shorter distances between the elements. With retardation the agreement between theory and experiment is remarkably good.