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On reflection from interfaces with some spatially dispersive metamaterials

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Abstract

Plane-wave reflection from interfaces with single and double wire media is considered. Such media exhibit strong spatial dispersion even at very low frequencies which causes appearance of additional waves. The problem of additional boundary conditions (ABC) in application to wire media is discussed and an ABC-free approach, known in solid state physics, is used. Expressions for the fields and Poynting vectors of refracted waves are derived. The directions and values of the power density flow of refracted waves are found and conservation of the power flow through the interface is checked.

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1. Introduction

The wire medium (WM) is an artificial medium formed by lattices of ideally conducting thin wires. Such a metamaterial is described usually at low frequencies as a uniaxial crystal, whose permittivity tensor components are expressed by the plasma model. It has been shown in Ref. [1], that if the wavevector in a WM has a nonzero component along the wires, the plasma model should be corrected introducing spatial dispersion (SD). Strong spatial dispersion is inherent to WM at any frequency, including the very large wavelength limit. This concerns both the usual (single) WM and double WM, formed by two mutually orthogonal wire lattices. Recently, the more complicated problem of double WM was solved in Ref. [2] numerically, in Ref. [3] using a semi-analytical approximation of the local field, and in Ref. [4] both numerically and using the effective medium (EM) approach. In the last paper a very good agreement between the results given by the EM and full-wave theories for all types of waves in double WM (if the wires are thin) has been demonstrated.

One of the most important effects of SD is the existence of additional waves and necessity of additional boundary conditions (ABC) for solution of any boundary value problem. In this paper, we consider a wave refraction problem at an interface of a semi-infinite WM. Both single WM and double WM are discussed.

In this paper, we consider plane-wave reflection and refraction at an interface of WM using the effective medium approach. We assume that the wire arrays are identical and are not connected (in double WM case), the lattice period is equal to L in the x, y, and in z (in double WM case) directions, and the radius of the wires $r_0 \ll L$. We assume also that the interface of the double WM lies in the (xy) plane, and the incident wave vector lies in the (yz) plane. In the single WM case we assume that the wires are normal to the interface (directed along z-axis). The effective medium model is used for description of the WM, and the ABC-free approach introduced in Ref. [5] is applied for overcoming the problem of additional waves. The reflection coefficient, field amplitudes and Poynting vectors for refracted waves are calculated.

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2. Eigenwaves in double wire medium

Assuming space-time dependence of fields as $e^{i(\omega t - k_y y - k_z z)}$, there are non-zero wave vector components parallel to the wires. Anisotropy appears in this electromagnetic crystal with a square lattice, and a double WM behaves as a biaxial crystal with the relative permittivity dyadic

$$\bar{\bar{\varepsilon}} = \varepsilon_x \mathbf{u}_x \mathbf{u}_x + \varepsilon_y \mathbf{u}_y \mathbf{u}_y + \varepsilon_z \mathbf{u}_z \mathbf{u}_z \quad (1)$$

with

$$\varepsilon_x = \varepsilon_{\rm h}, \quad \varepsilon_{y,z} = \varepsilon_{\rm h}\left(1 - \frac{k_{\rm p}^2}{k^2 - k_{y,z}^2}\right), \quad (2)$$

where $k = k_0\sqrt{\varepsilon_{\rm h}}$, $k_0 = \omega/c$, c is the speed of light, $\varepsilon_{\rm h}$ is the relative permittivity of the host medium, and $k_{\rm p}$ is the plasma wavenumber, calculated using approximate formula (see Ref. [1]).

For the single WM the permittivity component $\varepsilon_y = \varepsilon_{\rm h}$. Model (2) is valid both for real and imaginary k_y (propagating and evanescent waves, respectively) (see Ref. [1]).

Propagation constant k_y is determined by the incidence angle $k_y = k_0 \sin\theta$ and k_z is found from the dispersion equation, which has the following form for waves in double WM [4]:

$$T(k_z, \omega) = k_z^2 - \left[k^2 \varepsilon_y(k_y) - k_y^2 \frac{\varepsilon_y(k_y)}{\varepsilon_z(k_z)}\right] = 0. \quad (3)$$

Solutions for k_z is given in Ref. [4]. Two TM modes propagating or attenuating in both directions follow from the EM theory. Including the TE wave, there exist three waves. The conventional isotropic plasma model leads to only one wave for a certain direction and polarization, namely,

$$k_z = \sqrt{k_0^2 \varepsilon - k_y^2}, \quad \text{where } \varepsilon = \varepsilon_{\rm h}(1 - k_{\rm p}^2/k^2).$$

In the single WM case spatial dispersion also leads to two solutions with the same polarization:

$$k_{z+} = k \quad \text{(TEM mode)},$$
$$k_{z-} = \sqrt{k^2 - k_y^2 - k_{\rm p}^2} \quad \text{(TM mode)}. \quad (4)$$

It was demonstrated in Ref. [4], that electrodynamical calculations (using the three-dimensional Green's function) confirm the results of the effective medium theory with a high accuracy in a wide spectral range including the regions of evanescent and propagating waves.

3. Interface problem

Assuming the y-component of the electric field of the incident wave to be equal unity, and applying the continuity conditions of the tangential field components results in the reflection problem formulated as follows:

$$1 + R_E = E_+ + E_-,$$
$$(1 - R_E)/Z_0 = E_+/Z_+ + E_-/Z_-, \quad (5)$$

where R_E is the unknown reflection coefficient for the tangential component of electric field, E_+, E_- are the unknown amplitudes of refracted waves in wire medium, $Z_0 = \eta\cos\theta$, η is the wave impedance (TM) in free space and $Z_\pm$ are the wave impedances for the tangent field components of refracted waves. Thus the problem becomes similar to appearing in crystallooptics, where excitons arise and spatial dispersion cannot be neglected. It was pointed out first by Pekar [6], that the well-known Maxwell's boundary conditions (5) are not sufficient to connect the amplitudes of the incident and transmitted waves in adjoining media, if more than one independent wave can propagate in any of the contacting media.

In order to avoid the ABC problem we use an approach, proposed by Henneberger [5]. It is based on the assumption of an abrupt transition from medium to vacuum. It is assumed that the incident wave excites a source $s(z,\omega)$, located within a sub-surface layer. Applying this approach for considering our interface problem of free space and the wire medium, the wave equation for H_x in unbounded medium should be replaced by an inhomogeneous equation

$$\frac{\partial^2 H_x}{\partial z^2} + \left[k_0^2 \varepsilon_y(k_y) - k_y^2 \frac{\varepsilon_y(k_y)}{\varepsilon_z(k_z)}\right] H_x = s(z,\omega), \quad (6)$$

where H_x is the scattered field. It means that any propagating wave has to be created by a source. The proper source of the penetrating wave in the wire medium is the incident wave and the polarization induced by it in the medium. Such an externally controlled source can be identified with some polarization additionally induced to the one already described by $\bar{\bar{\varepsilon}}$. Therefore it is located only in the surface or in the transition region, where the induced polarization deviates from that in the bulk medium. After the Fourier transform of Eq. (6) one obtains

$$H_x(z,\omega) = \int_{-\infty}^{\infty} \frac{{\rm d}k_z}{2\pi} \frac{s(k_z,\omega) e^{ik_z z}}{T(k_z,\omega)}, \quad (7)$$

where $s(k_z,\omega)$ is the Fourier transform of $s(z,\omega)$. Assuming an abrupt transition from medium to vacuum, we can present a source as a delta function $s(z,\omega) = s_0(\omega)\delta(z)$, then $s(k_z,\omega) = s_0(\omega)$. If $T(k_z,\omega)$ is an analytical function, the integration in Eq. (7) can be performed using the residuum method. Residues can be found by presenting $1/T$ (Eq. (3)) in the form:

$$\frac{1}{T(k_z,\omega)} = \frac{\beta_+}{k_z^2 - k_{z+}^2} + \frac{\beta_-}{k_z^2 - k_{z-}^2}, \quad (8)$$

where the coefficients are

$$\beta_+ = \frac{k^2 - k_{\rm p}^2 - k_{z+}^2}{k_{z-}^2 - k_{z+}^2}, \quad (9)$$