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Multicritical phase diagrams of the ferromagnetic spin-$\frac{3}{2}$ Blume–Emery–Griffiths model with repulsive biquadratic coupling including metastable phases: The cluster variation method and the path probability method with the point distribution

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Abstract

We study the thermal variations of the ferromagnetic spin-$\frac{3}{2}$ Blume–Emery–Griffiths (BEG) model with repulsive biquadratic coupling by using the lowest approximation of the cluster variation method (LACVM) in the absence and presence of the external magnetic field. We obtain metastable and unstable branches of the order parameters besides the stable branches and phase transitions of these branches are investigated extensively. The classification of the stable, metastable and unstable states is made by comparing the free energy values of these states. We also study the dynamics of the model by using the path probability method (PPM) with the point distribution in order to make sure that we find and define the metastable and unstable branches of the order parameters completely and correctly. We present the metastable phase diagrams in addition to the equilibrium phase diagrams in the $(kT/J, K/J)$ and $(kT/J, D/J)$ planes. It is found that the metastable phase diagrams always exist at the low temperatures, which are consistent with experimental and theoretical works.

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1. Introduction

The spin-$\frac{3}{2}$ Blume–Emery–Griffiths (BEG) model has been paid much attention for many years because of their simplicity and exhibits a variety of multicritical phenomena accompanied with the onset of first- and second-order phase transitions. The spin-$\frac{3}{2}$ BEG model is defined by the Hamiltonian

$$\mathscr{H} = -J\sum_{\langle ij\rangle} S_i S_j - K\sum_{\langle ij\rangle} S_i^2 S_j^2 + D\left(\sum_i S_i^2\right), \tag{1}$$

where each S_i can take the values $\pm\frac{3}{2}$ and $\pm\frac{1}{2}$ and $<ij>$ indicates summation over all pairs of nearest-neighbor sites. J, K and D are the bilinear exchange, biquadratic exchange and crystal–field interactions, respectively.

The phase diagrams of the ferromagnetic spin-$\frac{3}{2}$ BEG model for $K/J>0$ have been studied and its phase diagrams have been presented by a variety of method such as renormalization-group (RG) methods [1], the effective field theory (EFT) [2], the Monte Carlo (MC) simulations and a density–matrix RG method [3]. An exact formulation of the model on a Bethe lattice was studied by using the exact recursion equations [4]. The ground-state phase diagrams of the spin-$\frac{3}{2}$ BEG model

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was worked out by Canko and Keskin [5]. On the other hand, the ferromagnetic spin-$\frac{3}{2}$ BEG model with the repulsive biquadratic coupling, i.e. $K/J<0$ is now a subject of intense study. For example, an early attempt to study the spin-$\frac{3}{2}$ BEG model with $K/J<0$ was made by Barretto and Bonfim [6], and Bakkali et al. [7] within the MFA and also the MC calculation, and the EFT, respectively. Barreto and Bonfim calculated only the phase diagrams for the ferromagnetic isotropic spin-$\frac{3}{2}$ BEG model and Bakkali et al. [7] also presented two phase diagrams: One for the ferromagnetic spin-$\frac{3}{2}$ BC model in which is the spin-$\frac{3}{2}$ Ising model with only J and D interactions, and the other for the ferromagnetic isotropic spin-$\frac{3}{2}$ BEG model that is the spin-$\frac{3}{2}$ Ising model with only J and K interactions. Tucker [8] studied the ferromagnetic spin-$\frac{3}{2}$ BEG model with $K/J<0$ by using the cluster variation method in pair approximation (CVMPA) and he only presented the phase diagrams of the spin-$\frac{3}{2}$ BC model and isotropic spin-$\frac{3}{2}$ BEG model for several values of the coordination number. Bakchich and Bouziani [9] presented the phase diagram of the model in the (kT/J, D/J) plane for only the two different values of K/J within an approximate RG approach of the Migdal–Kadanoff type. Ekiz et al. [10] investigated the ferromagnetic spin-$\frac{3}{2}$ BEG model on the Bethe lattice using the exact recursion equations and presented the phase diagrams in the (kT/J, K/J) plane for several values of D/J and in the (kT/J, D/J) plane for several values of K/J in the absence of an external magnetic field (H). Ekiz [11] extended the previous work, i.e. Ref. [10] for the presence of an external magnetic field. He presented one phase diagram in the (kT/J, H/J) plane for $K/J=-0.5$ and $D/J=1.0$ and the other phase diagram in the (kT/J, K/J) plane for $H/J=2.0$ and $D/J=0.5$. In both figures, he used the coordination number $q=3$, 4, 6 and 8. Recently, Pınar et al. [12] studied the ferromagnetic spin-$\frac{3}{2}$ BEG model and found four new phase diagram topologies for $H=0.0$ and one new phase diagram topology for $H\neq 0.0$ within the lowest approximation of the cluster variation method (LACVM).

In spite of these studies, the critical behavior of the ferromagnetic spin-$\frac{3}{2}$ BEG model with repulsive biquadratic interaction ($K<0.0$) has not been thoroughly explored. Especially, the metastable and unstable branches of the order parameters and their phase transitions were not studied. Moreover, metastable phase diagrams of the model were also not calculated. Whereas, the metastable phase diagrams of the spin-1 BEG model were investigated extensively for $K>0.0$ [13] and also $K<0.0$ [14]. Therefore, the purpose of this work is to investigate the behavior of the thermal variation of the order parameters in depth and to obtain the metastable and unstable branches of the order parameters and to examine their phase transitions for repulsive biquadratic interaction. We also study the dynamics of the model by using the path probability method (PPM) with the point distribution [15] in order to make sure that we find and classify the metastable and unstable branches of the order parameters completely and correctly. Finally, we present the metastable phase diagrams in addition to the equilibrium phase diagrams in the (kT/J, K/J) and (kT/J, D/J) planes. The LACVM, in spite of its limitations, is an adequate starting point. Within this theoretical framework it is easy to determine the complete phase diagrams and find the some outstanding features in the temperature dependencies of the order parameters and as well as obtain metastable portion of the phase diagrams. It also offers a very practical and simple tool to solve most collective phenomena.

The outline of the remaining part of this paper is as follows. In Section 2, we define the model briefly and obtain its solutions at equilibrium within the LACVM. The thermal variations of the system are investigated in Section 3. The dynamics of the model is studied by the PPM in Section 4. In Section 5, the transition temperatures are calculated precisely, and metastable phase diagrams are presented in addition to the equilibrium phase diagrams. Section 6 contains the summary and conclusion.

2. Model and method

The spin-$\frac{3}{2}$ BEG model is defined as a two-sublattice model with spin variables $S_i=\pm\frac{3}{2}, \pm\frac{1}{2}$ and $S_j=\pm\frac{3}{2}, \pm\frac{1}{2}$ on sites of sublattices A and B, respectively. The average value of each of the spin states will be denoted by X_1^A, X_2^A, X_3^A and X_4^A on the sites of sublattice A and X_1^B, X_2^B, X_3^B and X_4^B on the sites of sublattice B, which are also called internal or the state or point variables. X_1^A, X_1^B; X_2^A, X_2^B; X_3^A, X_3^B; X_4^A, X_4^B are the fractions of the spins that have the values $+\frac{3}{2}$, $+\frac{1}{2}$, $-\frac{1}{2}$and $-\frac{3}{2}$, respectively. These variables obey the following two normalization relations for A and B sublattices:

$$\sum_{i=1}^{4} X_i^A = 1 \quad \text{and} \quad \sum_{j=1}^{4} X_j^B = 1. \tag{2}$$

In order to account for the possible two-sublattice structure, we need six long-range order parameters, which are introduced as follows: $M_A \equiv \langle S_i^A \rangle$, $Q_A \equiv \langle (S_i^A)^2 \rangle$, $R_A \equiv \langle (S_i^A)^3 \rangle$, $M_B \equiv \langle S_j^B \rangle$, $Q_B \equiv \langle (S_j^B)^2 \rangle$, $R_B \equiv \langle (S_j^B)^3 \rangle$ for A and B lattices, respectively. $\langle \ldots \rangle$ denotes the thermal average, M_A and M_B are the average magnetizations which is the excess of one orientation over the other orientation, called magnetizations. Q_A and Q_B are the quadrupolar moments which are the average squared magnetizations, and R_A and R_B are the octupolar order parameters, that are the average cubed magnetizations, for A and B sublattices, respectively.