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# Phase diagram for magnetic nano-rings

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## Abstract

The minimum-energy single-domain magnetization state in a magnetized nano-ring is determined as a function of material and shape parameters. A phase diagram is derived within the framework of a Fourier-space approach for magnetic computations, showing the expected position of the ground state for any given set of external degrees of freedom. A series of micromagnetic simulations for suitably chosen parameters, show excellent agreement with the obtained theoretical results. An electron holography experiment has been carried out as a test on phase diagram reliability. The validity of the treatment, in particular the simplification employed in choosing ideal uniform rather than more physical quasi-uniform single-domain states, is thoroughly discussed in order to establish clear boundaries of applicability of the phase diagram.

*Keywords:* Magnetic nano-particles; Phase diagram; Magnetostatic energy; Micromagnetic simulations; Electron holography

## 1. Introduction

The study of magnetic nano-particles has seen a large research effort in recent years [1]. The main driving forces for this increased interest are ultra-high density magnetic storage devices [2], magnetic random access memory (MRAM) [3], ferrofluids [4], spintronics [5], magnetic semiconductors [6], nano-granular magnetic materials [7], and so forth. With this increased interest comes the realization that the shape of the nano-particles takes a center stage in determining their magnetic properties. It is well known (see for example Refs. [8,9]) that macroscopic magnets reduce their magnetostatic energy by subdividing into magnetic domains with differently oriented magnetic moments. This reduces the external fringing field and, hence, the magnetostatic energy of the system. When the system size is reduced, there will be a critical “size” below which the energy gained by reducing the fringing field no longer offsets the energy needed to introduce domain walls, so that a single domain state becomes more favorable. The

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word "size" has been highlighted because it is not always possible to simply state a single number as the single domain size limit. For spherical particles, one can define the size limit by a single number (the sphere radius), but for complex shapes a more advanced approach is needed, taking full account of the detailed aspects of the particle shape and of the corresponding shape anisotropy effects.

One of the main vehicles for the study of magnetic nano-particle systems is the micromagnetic simulation approach. There are several commercial and public domain packages available, and they have been used to study a wide variety of 2D and 3D shapes: triangles and squares, cubes, ellipsoids, cylinders, truncated pyramids, and so on (see Ref. [10] for an extensive overview). While the micromagnetics approach provides detailed information about the static and dynamic behavior of complex 2D and 3D magnetic systems, it is difficult (and time consuming) to perform a complete parameter study using a micromagnetic simulation. Such a study (e.g., determination of a magnetic phase diagram) would involve a systematic variation of all important shape and material parameters, which rapidly becomes unfeasible for complex multi-parameter shapes.

An alternative approach, the *analytical computation*, is often hampered by the need for substantial simplifications to the geometry and magnetization state of the system studied, so that the various magnetic quantities, such as the demagnetization factors, can actually be computed in closed form. This has been done for only a few highly symmetric shapes (e.g. sphere, ellipsoid [11], cylinder [12,13], and prism [14,15]). Recently, we have proposed a computational scheme (both numerical and analytical) to determine the complete demagnetization tensor field and the volumetric demagnetization factors for arbitrary shapes [16,13,17]. Once these quantities are available, it is often straightforward to determine the dependence of the energetics of the magnetic system on the shape parameters. This was done in analytical form for the interaction between two uniformly magnetized cylinders in Ref. [18], and a generalization of this analytical framework has been introduced in Ref. [19].

In the present paper, we undertake a systematic analytical investigation of the magnetic phase diagram of nano-rings. This structure has been recently proposed as a possible candidate for MRAMs [3], and studied to some extent by means of micromagnetic simulations combined with several experimental techniques [20–22]. In magnetic nano-rings, the central hole avoids the formation of a vortex core with axial magnetization [23], and dramatically improves the stability of the vortex state compared to nano-disks. The problem of magnetic phase diagrams of nano-particles is a long-standing problem in the theory of magnetism. A rigorous treatment would, in principle, be able to map the single- and multiple-domain states found in the nano-particle onto parameters of its geometry, material and size. We limit our treatment to the ring geometry, to single-domain size, uniaxial anisotropy, and we only consider three particular magnetization states for the ring to build the phase diagram upon: uniform in-plane, uniform axial, and vortex. The underlying assumption here is that the uniform magnetization state is a fair representation, in terms of energy, of the actual physical quasi-uniform ground state [24].

It is well known that the equilibrium magnetization configuration in non-ellipsoidal particles is never uniform. In fact, the demagnetization tensor field of any non-ellipsoidal shape is not uniform inside the body. Therefore, the non-uniform demagnetization field induces a non-uniform equilibrium magnetization distribution over the nano-ring. A vast amount of the literature is devoted to the determination, theoretical and/or experimental, of the actual ground state of non-ellipsoidal particles. It has been clearly established that for magnetized nano-rings, at least for a certain range of inner diameters, the single-domain in-plane ground state is the "onion" state [25]. For smaller inner radii, it is arguable that the ground state might be more similar to a "leaf" state, which is the expected ground state for in-plane magnetized disks in the region of parameter space where a vortex cannot nucleate [26]. No experimental evidence is currently available, to our best knowledge, for an axial quasi-uniform state when the ring is very thick. It might be argued from the