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Optimal design of magnetic systems

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Abstract

The methods and the algorithms of the magnetic system design, producing the fields with a required distribution (the inverse problem of magnetostatics), are considered. The mathematical statement of the problem is reduced to the expansion of a given function into Fourier series on a non-orthonormal basis. The basis functions are the fields of the sections of a magnetic system, and the coefficients of the expansion are the components of the magnetization vector of the sections (the vector synthesis). The algorithms proposed to solve the problem take into account the physical constraints on the magnetization distribution of the assembly sections.

The application of the vector synthesis guarantees the maximum possible efficiency of magnet utilization in an assembly. The combination of the vector synthesis and the geometrical one, at which the distribution of the magnetization of the assembly sections is fixed, but the optimal basis is selected, ensures the minimization of the mass of the magnets used.

The algorithms suggested for solving the synthesis problem are also effective in the presence of a ferromagnetic armature in the system.

The examples of syntheses of the magnetic systems, producing the plane–parallel and the axisymmetric fields under the constraints on the magnetization distribution, are given.

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0. Introduction

While stating a problem of a magnetic system (MS) design, the magnetic field properties are often expressed as a given distribution law in a working area (WA). The examples of such systems are the magnetic traps and lenses, the solenoids, the electric machines, the MS of a tomograph and many others. The design of the MS, producing a required field in a WA, is considered in the present paper.

The up-to-date MS is usually assembled from the homogeneously magnetized sections. At this, the MS sections may have different sizes and form (the geometrical parameters), and also different value and orientation of the magnetization.

There are at least two ways of MS design. From the mathematical point of view, both of them can be treated as an expansion of a given vector function, describing the field in a WA, into some non-orthonormal basis. At this, the basis functions are the fields of the sections of an assembled MS. The coefficients of optimal superposition of the basis functions for both approaches are the components of the magnetization vector of the MS sections.

One of the design methods is to find the optimal coefficients of an expansion of the required field into the fixed basis. This means physically that we fix the MS geometrical parameters (i.e. we fix the set of the basis functions) and seek the distribution of the magnetization of each MS section. We call such a way of optimal design of MS as *the vector synthesis*, because the desired coefficients of expansion are the magnetization vectors.

Another method of the MS design is to find an optimal set of the basis functions at fixed coefficients of the expansion. This means physically that we fix the

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magnetization distribution of each MS section and seek the optimal geometrical parameters of the sections. We call such a way of optimal design of MS as *the geometrical synthesis*.

Certainly, the combination of these two methods of MS design is also possible. We call the design method combining both the vector synthesis and the geometrical one as *optimization*.

There are several variants of stating the problem of optimal MS design, differing with the restrictions on its desired parameters. The following factors exert essential influence on the statement of the problem: the presence of the permanent magnets (PM) with various characteristics; the volume of the device, designated for the filling up by PM; and also the technological possibilities of MS manufacturing.

We call the magnetic system free from the restrictions specified above as *the ideal* one (IMS). Such MS produces in a working area the filed, best satisfying the requirements.

The practise of MS design has shown that IMS can produce any "reasonable" fields with very high accuracy at rather large number of sections.

The absence of the constraints on the MS volume and on the magnetization distribution does not allow creating IMS on practise. However, IMS allows to answer several important questions and to choose the directions of optimal MS searches, providing a satisfactory accuracy of the field reproduction in a WA at the fulfillment of the given constraints.

1. MS synthesis problem statement

MS synthesis problem is formulated as follows. *Given*: the geometrical parameters of WA V_{P}, and the distribution of the field in it $\vec{H}^0$. *To find*: the volume V_{M} and MS magnetization distribution $\vec{J}$, at which the field with a required distribution is produced in the WA (the parameters of IMS).

The mathematical statement of IMS synthesis problem is reduced to the integral equation of the first kind:

$$\frac{1}{4\pi}\int_{V_{\mathrm{M}}}\left[\frac{3\left(\vec{J}_{\mathrm{P}},\vec{r}_{\mathrm{P}Q}\right)\vec{r}_{\mathrm{P}Q}}{r_{\mathrm{P}Q}^5}-\frac{\vec{J}_{\mathrm{P}}}{r_{\mathrm{P}Q}^3}\right]\mathrm{d}V_{\mathrm{P}}=\vec{H}^0(Q), \tag{1}$$

where V_{M} is the MS volume (determined while the geometrical synthesis); $\vec{J}$ is the required magnetization distribution; $\vec{r}$ is the radius vector drawn from the point of integration to the point of observation; $\vec{H}^0$ is the given field in the WA.

The design problem is stated in the form (1), if it is known that it has an exact solution, the existence of which is determined by the right part $\vec{H}^0$ and the geometrical parameters of WA. Besides, the MS designed must be technically realizable. There we change the solution of the integral equation (1) to the minimization of the residual error norm in Lebesgue's space L_2:

$$\left\|\sum_P \vec{H}(P,Q)-\vec{H}^0(Q)\right\|=\min, \tag{2}$$

where $\vec{H}(P,Q)$ is the field of the P-th section in the point of observation Q.

The problem of MS design belongs to the class of the inverse problems of magnetostatics. While investigating the problems of such a class, there arise the questions of existence and uniqueness of a solution.

In this case, the uniqueness of the problem solution is not an excellent feature, because of the necessity to choose the most economically reasonable variant of MS. In practice, a designer is often forced to search several approximate solutions of a problem, which provide a satisfactory reproduction of the field in a WA. In other words, only an optimal MS, which produces the field with a satisfactory accuracy, and at this the costs of its production and operation are minimal, is practically acceptable.

According to [1], we say that Eq. (1) has approximate pseudosolution, if there is a set of such functions $\vec{J}_k \in L_2(V_{\mathrm{M}})$ that

$$\lim_{k\to\infty}\left\|\vec{H}_0-\frac{1}{4\pi}\int_{V_{\mathrm{M}}}\left[\frac{3\left(\vec{J}_k,\vec{r}\right)\vec{r}}{r^5}-\frac{\vec{J}_k}{r^3}\right]\mathrm{d}V\right\|=q, \tag{3}$$

where q is the lower bound of the residual error set.

It was shown in [2] that the necessary and the sufficient condition of the existence of an approximate pseudosolution of (1) reads as

$$\frac{1}{4\pi}\int_{V_{\mathrm{W}}}\left[\frac{3\left(\vec{H}^0,\vec{r}\right)\vec{r}}{r^5}-\frac{\vec{H}^0}{r^3}\right]\mathrm{d}V\neq 0, \tag{4}$$

where V_{W} is the WA volume; $\vec{H}^0$ is the given field in the WA; $\vec{r}$ is the radius vector drawn from the point of integration to the point of observation.

Condition (4) can be always satisfied on practice. This follows from the fact that the left part of Eq. (4) is the field of a fictitious PM with the magnetization distribution coinciding with a required distribution of the field in the WA. In practice, the given field always differs from identical zero. This means that integral (4) also differs from identical zero in the space surrounding the WA and, consequently, in the MS volume, if, certainly, it is not withdrawn on physically infinitely large distance from the WA. Thus, the condition of the existence of an approximate pseudosolution of Eq. (4) is always fulfilled.

However, it should be kept in mind that correct description of the field is often a complicated problem. And if it was not carried out in a proper way, then a satisfactory solution of the MS design problem cannot be obtained.

The problem of an optimal MS design with a minimal number of constraints is stated as follows. *Given*: the geometrical parameters of the WA and the distribution of