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A simple noise correction scheme for diffusional kurtosis imaging

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ABSTRACT

Purpose: Diffusional kurtosis imaging (DKI) is sensitive to the effects of signal noise due to strong diffusion weightings and higher order modeling of the diffusion weighted signal. A simple noise correction scheme is proposed to remove the majority of the noise bias in the estimated diffusional kurtosis.
Methods: Weighted linear least squares (WLLS) fitting together with a voxel-wise, subtraction-based noise correction from multiple, independent acquisitions are employed to reduce noise bias in DKI data. The method is validated in phantom experiments and demonstrated for in vivo human brain for DKI-derived parameter estimates.
Results: As long as the signal-to-noise ratio (SNR) for the most heavily diffusion weighted images is greater than 2.1, errors in phantom diffusional kurtosis estimates are found to be less than 5 percent with noise correction, but as high as 44 percent for uncorrected estimates. In human brain, noise correction is also shown to improve diffusional kurtosis estimates derived from measurements made with low SNR.
Conclusion: The proposed correction technique removes the majority of noise bias from diffusional kurtosis estimates in noisy phantom data and is applicable to DKI of human brain. Features of the method include computational simplicity and ease of integration into standard WLLS DKI post-processing algorithms.

1. Introduction

The magnitude reconstruction typically used in MRI combines the real and imaginary components of a complex spatial signal so that the resulting image is independent of the arbitrary phase setting and is not affected by phase artifacts [1]. However, this non-linear operation transforms the Gaussian (normal) distribution of random noise for the real and imaginary component images and thereby introduces a positive bias for the magnitude signal that is inversely related to the signal-to-noise ratio (SNR) [2,3].

For many MRI sequences, the SNR is sufficiently high so that this noise bias is negligible. However, several quantitative imaging techniques, such as diffusion, perfusion, and functional MRI, utilize echo-planar imaging (EPI), which has relatively poor SNR [4]. For diffusion MRI, the SNR is further reduced by diffusion weighting of the signal [5]. As a result, diffusion weighted images (DWIs) and diffusion parameters estimated from DWIs are particularly susceptible to the effects of noise [6]. For example, noise bias may significantly alter estimates for both the mean diffusivity (MD) and fractional anisotropy, as conventionally obtained with diffusion tensor imaging (DTI) [6–9].

Diffusional kurtosis imaging (DKI) is an increasingly utilized diffusion MRI technique, which extends DTI by relaxing the assumption that the displacement of water via diffusion in biological tissue follows a Gaussian distribution (NB: the distribution of water displacements should not be confused with the noise distribution) [10,11]. As a result, DKI includes the diffusional kurtosis in the DWI signal model through the 4th order cumulant expansion of the displacement distribution [10,11]. In order to more accurately characterize the DWI signal from restricted diffusion that occurs in biological tissues [12], DKI employs a more complicated signal model that includes higher order terms in the b-value, which typically requires the use of larger maximum b-values [10,11,13]. As a result, parameter estimation for DKI can be more sensitive to noise effects than for DTI [14–17], which limits the attainable resolution. Furthermore, certain regions of interest (ROIs) in the brain, such as the globus pallidus, have a relatively low SNR due to a high tissue iron concentration that shortens T2 [18]. As a result, noise disproportionately affects diffusion parameter estimation in these regions. It may therefore be beneficial to implement a noise correction scheme that can remove the majority of noise bias from DKI data and thereby improve the accuracy of DKI-derived diffusion parameter estimates.

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Noise correction is complicated by the fact that the variance of noise is not known a priori and must typically be estimated from characteristics of the reconstructed images [19–21]. Two methods that have been used previously involve calculation of the mean or standard deviation of background noise level or calculation of the variance of the image intensity over a spatial ROI [19–25]. Although the noise bias can theoretically be estimated by the mean or standard deviation of the background noise [19–21], this method is problematic for noise correction in DKI because EPI sequences are susceptible to image artifacts in the background region [26,27], which can lead to an overestimation of the noise bias. Measuring the variance of intensity values within a spatial ROI may also be unreliable, because a spatially homogeneous ROI may be difficult or impossible to find and intrinsic variability in the spatial signal also leads to an overestimation of the noise bias. As a result, voxel-wise noise estimations from multiple, independent acquisition schemes are preferred over single acquisition methods [22,25]. In addition, ROI-based noise estimates are not suitable for spatially varying noise distributions, as may result from the use of parallel imaging, which also requires the noise level to be estimated in a voxel-wise manner [28–31]. Since parallel imaging is routinely used to decrease signal acquisition time, background noise estimators are almost never preferred to voxel-wise methods. The major drawbacks to multiple acquisition schemes are increased scan time and increased signal variance from non-thermodynamic factors such as patient motion or pulsatile changes.

There are a wide variety of noise correction methods available, including those that utilize lookup tables [3], global correction via a single noise correction term [19–21], non-linear maximum likelihood (ML) or maximum a posteriori (MAP) estimation which utilize Bessel functions in the measured signal model [12,25,32], nonlocal maximum likelihood estimation which sample neighboring voxels to determine spatially varying noise statistics [33,34], nonlinear diffusion filtering for spatially varying noise [35], spatially varying noise corrections involving confluent hypergeometric functions [31,36], and multistep weighting strategies to remove errors that can be introduced through the estimated weight factors [37]. Although the more complicated noise correction techniques may have desirable theoretical properties under certain circumstances, they can be difficult to implement, require nonlinear optimization, and/or involve iterative fitting algorithms, which can be computationally demanding and may not guarantee a unique solution. DKI is well suited for linear estimation techniques because the system of equations can readily be linearized; however, ML estimation has also been applied to DKI data with spatially varying noise levels for robust parameter estimation [12]. For a comprehensive discussion on the effects of different fitting strategies in diffusion MRI, the reader is directed to the work of Veraart et al. [37,38].

The goal of this work is to describe a simple method for noise correction which is both straightforward to implement and removes the majority of the noise bias for DKI with low SNR. Our approach takes into account both the noise bias of the DWI signal, as well as the dependence of the noise variance on the signal magnitude that results from the Rician distribution [2,3,21]. We consider a weighted linear least squares (WLLS) DKI analysis and propose a new set of weight factors to reduce the noise bias in the DKI parameter estimates. These weight terms account for the varying degrees of noise variance in the measured diffusion signal assuming the noise in each independent scan is well-described by a Rician distribution. The noise correction terms are calculated from a voxel-wise noise map estimated from multiple image acquisitions, thereby accommodating for spatially varying noise [31]. This noise correction scheme is compatible with well-established DKI analysis procedures [13]. By preserving the linearity of the estimation problem, our approach guarantees a unique solution with minimal computational demands.

2. Theory

2.1. Noise correction in diffusion MRI

We consider first the DKI signal model for an individual voxel:

$$S(\boldsymbol{g}) = S_0 e^{-bD(\boldsymbol{n})+\frac{1}{6}b^2D^2(\boldsymbol{n})K(\boldsymbol{n})}, \tag{1}$$

where $S(\boldsymbol{g})$ is the theoretical diffusion signal for a diffusion encoding gradient vector, $\boldsymbol{g}$, $S_{\boldsymbol{0}}$ is the diffusion signal with no diffusion weighting, and $D(\boldsymbol{n})$ and $K(\boldsymbol{n})$ are the directional diffusivity and the diffusional kurtosis, respectively, along the normalized vector $\boldsymbol{n} = \boldsymbol{g}/\sqrt{b}$ [10,11]. Here we have defined $\boldsymbol{g} = \sqrt{b}\boldsymbol{n}$, with b being the b-value which summarizes the stength of the diffusion weighting and $\boldsymbol{n}$ being the normalized diffusion encoding gradient direction.

The signal model in Eq. (1) describes the fourth-order cumulant expansion of the diffusion signal, which accounts for the leading effects of restricted, non-Gaussian diffusion on the diffusion MR signal, provided the b-value is not too large. However, in magnitude MR images, the measured signal is a biased estimator of the theoretical signal, and the expected value of the measured signal is always greater than the expected value of the theoretical signal [3]. The difference between these two quantities is termed the noise bias.

If we consider Rician distributed noise and, for the moment, assume the noise variance, σ^2, in the channels of the receive coil is known, then the expected value of the measured signal squared for arbitrary SNR is given by [2,19,20]:

$$\left\langle M^2 \right\rangle = \left\langle S^2 \right\rangle + 2\sigma^2, \tag{2}$$

where the brackets denote the expected value of a random variable and M and S refer to the measured and theoretical signals, respectively.

Eq. (2) demonstrates a few key features of the effects of noise in magnitude MR images. First, noise adds in quadrature, and so in applying noise correction it is natural to consider the squares of the measured and ideal signals, sometimes termed power images [19]. Second, the variance of noise, σ^2, does not depend on the signal amplitude or the applied b-value. Nonetheless, the noise bias in the root mean square (RMS) measured signal estimator will depend on the signal amplitude because of the nonlinearity of Eq. (2), which is the source of systematic bias in diffusion parameter estimates taken from noisy DWIs. Consequently, for low b-value (high SNR) images the theoretical signal is typically sufficiently strong so that noise bias is negligible. However, for high b-value (low SNR) images, the noise bias accounts for a larger portion of the measured signal, and it becomes important to account for the effects of noise in the data analysis.

During a diffusion MR experiment, we acquire voxel-wise estimates for the measured signal in Eq. (2) over a pre-determined set of diffusion weighting gradient vectors. The expected value for the noise corrected signal squared for a given gradient vector can then be estimated by [19,20]:

$$\left\langle S_{\boldsymbol{g}}^2 \right\rangle = \left\langle M_{\boldsymbol{g}}^2 \right\rangle - \eta^2, \tag{3}$$

where we have defined the noise parameter term, $\eta^2 = 2\sigma^2$, and the subscript $\boldsymbol{g}$ denotes the applied diffusion weighting gradient vector, $\boldsymbol{g}$. We have converted the variable $\boldsymbol{g}$ from a functional argument in