Magnetic Resonance Imaging 33 (2015) 174–179

Contents lists available at ScienceDirect

Magnetic Resonance Imaging

journal homepage: www.mrijournal.com

Improving synthesis and analysis prior blind compressed sensing with low-rank constraints for dynamic MRI reconstruction

Angshul Majumdar *

Indraprastha Institute of Information Technology Electronics and Communications Engineering, Phase III, Okhla Industrial Complex, New delhi, India

ARTICLE INFO

Article history:
Received 10 March 2014
Revised 14 April 2014
Accepted 25 August 2014

Keywords:
Blind compressed sensing
Split Bregman
Dynamic MRI reconstruction
Matrix recovery

ABSTRACT

In blind compressed sensing (BCS), both the sparsifying dictionary and the sparse coefficients are estimated simultaneously during signal recovery. A recent study adopted the BCS framework for recovering dynamic MRI sequences from under-sampled K-space measurements; the results were promising. Previous works in dynamic MRI reconstruction showed that, recovery accuracy can be improved by incorporating low-rank penalties into the standard compressed sensing (CS) optimization framework. Our work is motivated by these studies, and we improve upon the basic BCS framework by incorporating low-rank penalties into the optimization problem. The resulting optimization problem has not been solved before; hence we derive a Split Bregman type technique to solve the same. Experiments were carried out on real dynamic contrast enhanced MRI sequences. Results show that, with our proposed improvement, the reconstruction accuracy is better than BCS and other state-of-the-art dynamic MRI recovery algorithms.

1. Introduction

In this work, we address the problem of reconstructing a dynamic MRI sequence from its under-sampled K-space frames. The data acquisition is expressed as follows:

$$y_t = R_t F x_t + \eta,\ \eta \sim N\left(0, \sigma^2\right) \tag{1}$$

where x_t denotes the t^{th} frame to be reconstructed, F is the Fourier transform, R_t is the K-space sampling mask for the said instant, y_t is the acquired K-space samples and η is the noise.

Assuming that there are T such frames, (1) can be compactly represented as:

$$vec(Y) = \Phi vec(X) + \eta \tag{2}$$

where $Y = [y_1| \ldots |y_T]$, $X = [x_1| \ldots |x_T]$ and $\Phi = BlockDiag(R_t F)$.

The problem is to recover, X given Y and Φ. Usually compressed sensing (CS) [1–3] based techniques are employed to recover them. CS exploits the spatio-temporal redundancy of the sequence X in order to recover it. The spatio-temporal redundancy leads to sparsity in a transform domain, and CS techniques utilize this sparsity for recovery.

There is an alternate reconstruction approach that departs from standard CS techniques. The dynamic MRI sequence X is low-rank. This is because the frames are temporally correlated, and hence the columns of X are not independent. Based on this argument, it was shown [4] that low-rank matrix recovery techniques can be employed to recover the dynamic MRI sequence. Unfortunately, this method cannot compete with CS based reconstruction techniques in terms of accuracy.

Some recent studies proposed combining CS based approaches with low-rank recovery techniques [5–7]. These papers showed that, such combined approaches yield better results than using sparsity based techniques or low-rank recovery techniques individually.

Recently blind compressed sensing (BCS) formulation was proposed [8]. CS assumes that the sparsifying basis is known a priori. BCS argues that, knowing the sparsifying basis is not necessary; it is possible to estimate the basis and the sparse coefficient simultaneously. Since the sparsifying basis is unknown; hence the name 'Blind'. It was shown in [9] that BCS can be used for dynamic MRI reconstruction.

The BCS technique do not explicitly incorporate the fact that the MRI sequence is low-rank; as mentioned before, exploiting this property had shown better reconstruction previously [5–7]. In this work, we propose to incorporate the low-rank property in order to improve the BCS recovery results.

The rest of the work is organized into several sections. Previous work in dynamic MRI reconstruction will be briefly discussed in the following section. Our proposed methodology is described in Section 3. The experimental results will be in Section 4. Finally, the conclusions of this work will be discussed in Section 5.

* Tel.: +919958164278.
E-mail addresses: angshul@iiitd.ac.in, angshulm@ece.ubc.ca.

http://dx.doi.org/10.1016/j.mri.2014.08.031
0730-725X/

2. Review of literature

The most general representation for the dynamic MRI reconstruction problem is expressed in (2). Compressed sensing (CS) based techniques exploit the spatio-temporal redundancy of the sequence for reconstruction. It is well known that MR images (columns of X) are sparse in wavelet domain. Since the sequence is temporally correlated, the variation along the rows of X can be assumed to be smooth and hence is likely to have a compact representation in the Fourier domain. In [1,2] the following formulation was proposed for recovering the sequence:

$$\min_X \|vec(Y)-\Phi vec(X)\|_2^2+\lambda\|W\otimes F_{1D}vec(X)\|_1 \quad (3)$$

Here the W is the wavelet transform to sparsify along the spatial direction, and F_{1D} is the one dimensional Fourier transform to sparsity along the temporal direction. The Kronecker product is a convenient notation for this expression.

In [3] it was shown that one can also recover the sequence by only accounting for the temporal difference as follows:

$$\min_X \|vec(Y)-\Phi vec(X)\|_2^2+\lambda TV_t(X) \quad (4)$$

where $TV_t=\sum ||\nabla_t X_i||_1$ and ∇_t denotes the temporal differentiation for the i^{th} pixel.

Here the argument is that since the frames are temporally correlated, the difference between the frames is sparse, and this sparsity can be exploited for recovery.

Departing from CS based techniques, it was shown by [4] that, the matrix X can also be represented as a low-rank matrix. The argument is simple—since the frames are correlated, the columns of X are not linearly independent. In [4] a matrix factorization based technique was used for solving the recovery problem; however other techniques like nuclear norm minimization can be used as well.

More recent studies [5–7] proposed combining CS with low-rank matrix recovery. The following optimization problem is used for reconstructing X:

$$\min_X \|vec(Y)-\Phi vec(X)\|_2^2+\lambda_1\|\Psi_S\otimes\Psi_T vec(X)\|_1+\lambda_2\|X\|_* \quad (5)$$

Here Ψ_S and Ψ_T are transformed to sparsify along the spatial and temporal directions. In [5] these are respectively spatial and temporal finite differencing; in [6,7] they are wavelet and Fourier. The nuclear norm penalty ($||X||_*$) enforces a low-rank solution. The two parameters—λ_1 and λ_2—balances the relative importance of the sparsity and the low-rank penalties.

So far, we have been discussing techniques where the sparsifying transform (wavelet, Fourier, finite differencing etc.) is known. A recent work [8] showed that instead of using fixed sparsifying basis, better results can be obtained if a learned basis was employed. Here dictionary learning techniques were employed to estimate the sparsifying dictionary from the training data. The learned dictionary was finally used for actual dynamic MRI reconstruction. They showed that, such a learned dictionary based reconstruction yields considerably better results than previous CS based recovery methods that used fixed sparsifyng basis.

It should be noted that the prior work [8] had two phases: training—where the dictionary is estimated/learned; and testing—where the learned dictionary is employed for dynamic MRI reconstruction. The blind compressed sensing (BCS) [9] formulation marries the two phases—in BCS, both the empirical sparsifying dictionary and the sparse coefficients are estimated simultaneously during signal recovery.

In BCS, the signal is assumed to be sparse in an unknown basis, i.e. $X=DZ$ where D is the sparsifying basis and Z is the sparse coefficient set. The BCS formulation for dynamic MRI [10] is as follows:

$$\min_{D,Z} \|vec(Y)-\Phi vec(DZ)\|_2^2+\lambda_1\|Z\|_1+\lambda_2\|D\|_F^2 \quad (6)$$

Obviously this is not a convex problem since the unknowns D and Z are in a product form (a bilinear problem). However, it has been shown in [10] that this technique yields better results than low-rank recovery techniques [4]. However, it is known that simple low-rank recovery techniques do not yield the best reconstruction results. Thus an improvement over such a technique do not mean much; one does not know if BCS can compete with state-of-the-art techniques which combine sparsity with rank deficiency, e.g. k-t SLR [5].

The BCS technique we discussed [9,10] are examples of sparse synthesis prior. A co-sparse analysis prior BCS can also be formulated [11]. The difference between synthesis and analysis prior is that the later assumes DX to be co-sparse; D being the dictionary and X being the signal of interest. The analysis prior BCS was not used for MRI reconstruction; it was used for image denoising. If this technique is adopted for dynamic MRI recovery, the optimization problem would be,

$$\min_{D,X} \|vec(Y)-\Phi vec(X)\|_2^2+\lambda_1\|DX\|_1+\lambda_2\|D\|_F^2 \quad (7)$$

In CS based MRI reconstruction, it has been observed repeatedly that the analysis prior yields better recovery results compared to the synthesis prior [12,13]. We expect that similar improvements can be achieved for BCS as well.

3. Exploiting rank-deficiency in BCS

As mentioned before, the analysis prior BCS have not been applied for dynamic MRI reconstruction. Hence it would be interesting to see how it performs. However, this may not be a significant improvement. What is even more interesting is to follow cue from prior studies [5–7] that combined rank-deficiency with sparsity based techniques. In this work, we propose to exploit the low-rank property of the MRI sequence X within the BCS reconstruction framework. This can be achieved by adding low-rank penalties to (6) and (7), leading to:

$$\min_{D,Z} \|vec(Y)-\Phi vec(DZ)\|_2^2+\lambda_1\|Z\|_1+\lambda_2\|Z\|_*+\lambda_3\|D\|_F^2 \quad (8)$$

$$\min_{D,X} \|vec(Y)-\Phi vec(X)\|_2^2+\lambda_1\|DX\|_1+\lambda_2\|X\|_*+\lambda_3\|D\|_F^2 \quad (9)$$

The low-rank penalty on the signal X is obvious for the co-sparse analysis prior formulation (9); since the MRI sequence is low-rank, we impose the penalty on X. For the synthesis prior (8), one might ask, why the low-rank penalty is imposed on Z. This too is simply to explain—BCS minimizes the Frobenius norm of the dictionary, hence D cannot be of low-rank; the only possibility is to impose a low-rank penalty on Z.

These formulations (8) and (9) are not convex, but then none of the BCS formulations are. Moreover, there are no algorithms to solve (8) and (9), because such problems have not been encountered before. In the following sub-section, we propose to derive efficient algorithms to solve these in the following section.

The reviewer pointed out that usually dynamic MRI sequences are acquired by multi-coil scanners. It is easy to incorporate the SENSE framework [14] into our proposed formulation. In SENSE, the data acquisition from each channel (c) is expressed as:

$$y_c=RFS_c x+\eta \quad (10)$$