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Adaptive fixed-point iterative shrinkage/thresholding algorithm for MR imaging reconstruction using compressed sensing

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ABSTRACT

Recently compressed sensing (CS) has been applied to under-sampling MR image reconstruction for significantly reducing signal acquisition time. To guarantee the accuracy and efficiency of the CS-based MR image reconstruction, it necessitates determining several regularization and algorithm-introduced parameters properly in practical implementations. The regularization parameter is used to control the trade-off between the sparsity of MR image and the fidelity measures of *k*-space data, and thus has an important effect on the reconstructed image quality. The algorithm-introduced parameters determine the global convergence rate of the algorithm itself. These parameters make CS-based MR image reconstruction a more difficult scheme than traditional Fourier-based method while implemented on a clinical MR scanner. In this paper, we propose a new approach that reveals that the regularization parameter can be taken as a threshold in a fixed-point iterative shrinkage/thresholding algorithm (FPIST) and chosen by employing minimax threshold selection method. No extra parameter is introduced by FPIST. The simulation results on synthetic and real complex-valued MRI data show that the proposed method can adaptively choose the regularization parameter and effectively achieve high reconstruction quality. The proposed method should prove very useful for practical CS-based MRI applications.

1. Introduction

In recent years, compressed sensing (CS) has attracted much attention in many areas, especially in signal processing, by offering the possibility of accurately recovering a sparse or compressible signal from fewer measurements than that suggested by the conventional Nyquist sampling theory [1–3]. Due to the implicit sparsity of MR images and the data acquisition mode of MRI meet the requirements of CS theory [4–7], CS-based MR image reconstruction has the potential to reduce scan time considerably while keeping the images of high quality.

Applying CS to MR image reconstruction from under-sampled *k*-space data, one seeks to solve the following ℓ_1-norm minimization problem [4]:

$$\min_{x\in\mathbb{C}^n} \| F_u x - y\|_2^2 + \Lambda\|\Psi \mathrm{x}\|_1, \tag{1}$$

where x is the reconstructed image, y is the measured *k*-space data acquired by an MR scanner, n is the number of pixels of x and λ is the regularization parameter. F_u and Ψ denote the under-sampling Fourier transform and the sparsifying transform respectively. There are several algorithms that have been proposed in the literature to find the optimal solution for the minimization problem (1), such as non-linear conjugate gradient algorithm (NLCG) [4], two-step iterative shrinkage/thresholding algorithm (TwIST) [8], and fast iterative shrinkage/thresholding algorithm (FISTA) [9], etc. In order to implement these algorithms for CS-based MR image reconstruction, several parameters should be determined properly in advance. One of them is the regularization parameter λ, which controls the trade-off between the data fidelity of the reconstruction to the measurements and the sparsity of MR images [4]. Proper selection of λ is important in guaranteeing the reconstructed image quality. Others are imposed by the algorithms themselves, such as the parameter L in FISTA which plays the role of a step size related to the convergence rate and should be larger than an unknown Lipschitz constant. These parameters make it much difficult to implement reliable and fast CS-based MR image reconstruction on clinical MR scanners.

There are two ways to determine the value of λ. One is to solve (1) with different values of λ and select the one which satisfies $||F_u x - y||_2 \approx e$, where e controls the upper bound of the recovery error [4]. Unfortunately, e is usually unknown previously. The other is to extend existing parameter choice methods [10], proposed for Tikhonov regularization problems, to (1) for finding out the optimal

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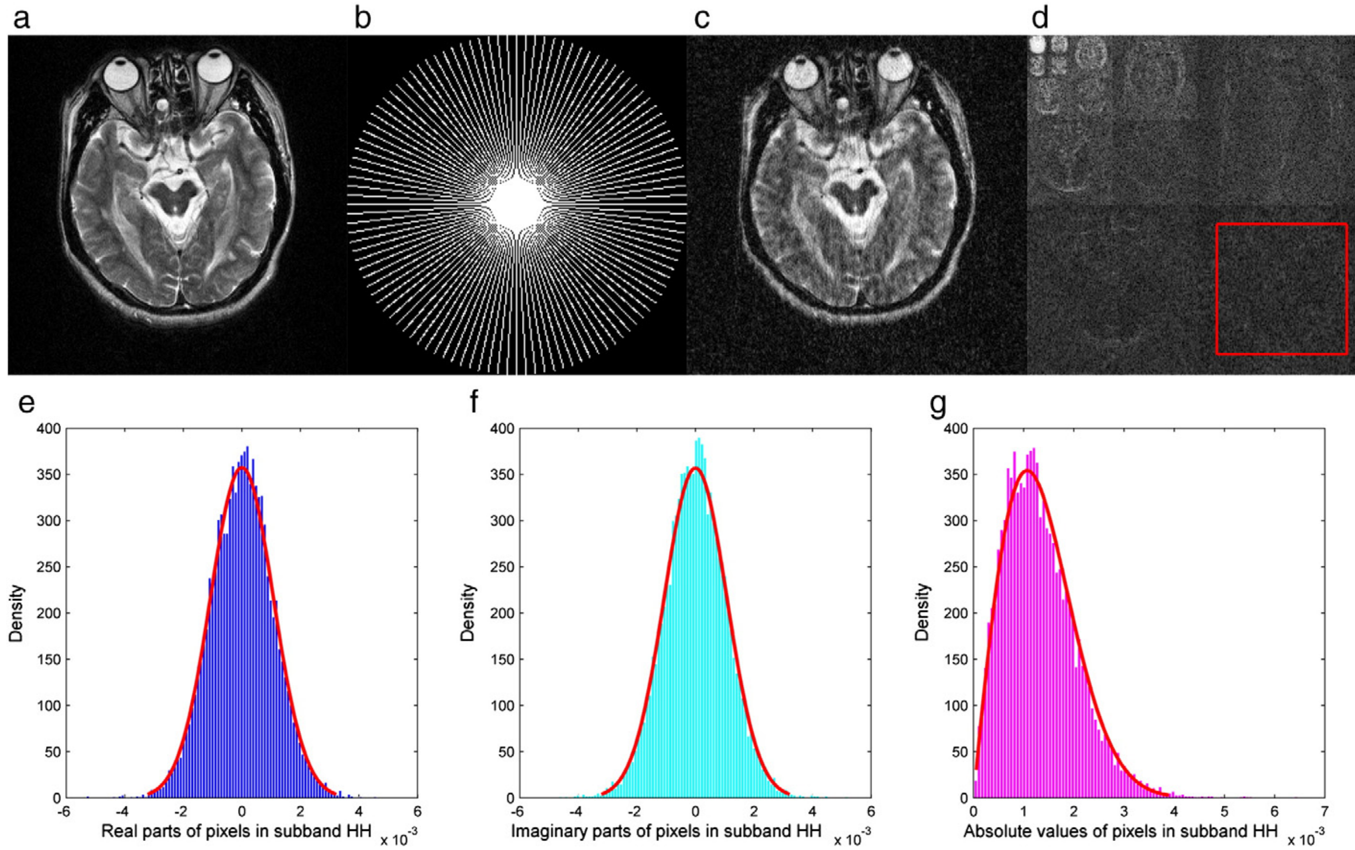

Fig. 1. Data distribution in the highest sub-band HH of wavelet transform of 80% under-sampled MR image: (a) MR image reconstructed from fully sampled data by inverse FFT; (b) 2D radial under-sampling scheme; (c) MR image reconstructed from 80% under-sampled data by zero-padded inverse FFT; (d) wavelet transform of under-sampled MR image, the ROI region contains the pixels of highest-frequency sub-band; (e) probability density distribution of real parts of pixels in highest-frequency sub-band; (f) probability density distribution of imaginary parts of pixels in highest-frequency sub-band; (g) probability density distribution of absolute values of pixels in highest-frequency sub-band.

value of the regularization parameter as suggested in Ref. [3]. However, most popular parameter choice approaches used in practical problems, such as Generalized Cross-Validation (GCV) [11] and L-curve (LC) [12], need to solve the target regularization problem many times to find the best regularization parameter that meets their criteria. In CS-based MR image reconstruction, almost all existing algorithms for solving (1) employ iterative schemes [4,5,8,9,13,14], which make slow processes to find the final solution. Thus it will be very numerically costly while employing these parameter choice methods to determine the regularization parameter in (1). Algorithm-introduced parameters are usually estimated empirically in advance. So the decision of the proper selection of these parameters is very important for practical CS-based MRI applications.

In this paper, we present a fixed-point iterative shrinkage/thresholding algorithm (FPIST) to solve the optimization problem (1) for under-sampling MR image reconstruction. No extra parameters are introduced in FPIST. Based on this algorithm, the regularization parameter λ is regarded as a threshold and chosen by estimating background noise level in wavelet domain. We evaluate the performance of the proposed method by simulations based on synthetic complex-valued data.

The remaining parts of this article are organized as follows: Section 2 describes the proposed algorithm with adaptive regularization parameter selection method in details, and gives the framework of the proposed approach. Section 3 shows the simulation results from a numerical phantom and in vivo MRI data, and followed by the study conclusion in Section 4.

2. Methods

2.1. Fixed-point iterative shrinkage/thresholding algorithm

Take the orthogonal wavelets as the sparsifying transform Ψ, and define

$$z \equiv \Psi x, \tag{2}$$

$$F \equiv F_u + F_{\overline{u}}, \tag{3}$$

$$R_{(u,\overline{u})} \equiv F_{(u,\overline{u})}\Psi^{-1}, \tag{4}$$

where the subscript u indicates that the under-sampling operation is applied, while the subscript $\overline{u}$ denotes its complementary operation. F is the normalized full-sampling Fourier transform. Rewrite (1) as follows:

$$\min_{z\in\mathbb{C}^n} \|R_u z - y\|_2^2 + \Lambda\|z\|_1. \tag{5}$$

Since that $R_u = R - R_{\overline{u}}$ and y is the under-sampled k-space data, we obtain

$$\min_{z\in\mathbb{C}^n} \|Rz - y\|_2^2 - \|R_{\overline{u}} z\|_2^2 + \Lambda\|z\|_1. \tag{6}$$

The unique minimizer of (6) satisfies

$$R^H Rz - R^H y - R_{\overline{u}}^H R_{\overline{u}} z + \frac{\Lambda}{2}\gamma(z) = 0, \tag{7}$$