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Noise estimation in parallel MRI: GRAPPA and SENSE

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ABSTRACT

Parallel imaging methods allow to increase the acquisition rate via subsampled acquisitions of the **k**-space. SENSE and GRAPPA are the most popular reconstruction methods proposed in order to suppress the artifacts created by this subsampling. The reconstruction process carried out by both methods yields to a variance of noise value which is dependent on the position within the final image. Hence, the traditional noise estimation methods – based on a single noise level for the whole image – fail. In this paper we propose a novel methodology to estimate the spatial dependent pattern of the variance of noise in SENSE and GRAPPA reconstructed images. In both cases, some additional information must be known beforehand: the sensitivity maps of each receiver coil in the SENSE case and the reconstruction coefficients for GRAPPA.

1. Introduction

Magnetic Resonance Imaging (MRI) is known to be affected by several sources of quality deterioration, due to limitations in the hardware, scanning times, movement of patients, or even the motion of molecules in the scanning subject. Among them, noise is one source of degradation that affects acquisitions. The presence of noise over the acquired MR signal is a problem that affects not only the visual quality of the images, but also may interfere with further processing techniques such as registration or tensor estimation in Diffusion Tensor MRI [1].

Noise has usually been statistically modeled attending to the scanner coil architecture. For a single-coil acquisition, the complex spatial MR data are typically modeled as a complex Gaussian process, where the real and imaginary parts of the original signal are corrupted with uncorrelated Gaussian noise with zero mean and equal variance σ_n^2. Thus, the magnitude signal is the Rician distributed envelope of the complex signal [2]. This Rician distribution whose variance is the same for the whole image is also known as *homogeneous* Rician distribution or, more accurately, *stationary* Rician distribution, and it has been the most used model in literature for multiple applications [3–8].

When a multiple-coil MR acquisition system is considered, the Gaussian process is repeated for each receiving coil. As a consequence, noise in each coil in the **k**-space can be also modeled as a complex stationary Additive White Gaussian Noise process, with zero mean and equal variance. In that case, the noise in the complex signal in the **x**-space for each coil will also be Gaussian. If the **k**-space is fully sampled, the composite magnitude signal (CMS, i.e. the final real signal after reconstruction) is obtained using methods such as the sum-of-squares (SoS) [9]. Assuming the noise components to be identically and independently distributed, the CMS will follow a non-central chi (nc-χ) distribution [9]. If the correlation between coils is taken into account, the data do not strictly follow an nc-χ but, for practical purposes, it can be modeled as such, but taking into account effective parameters [10,11].

However, in multiple-coil systems, fully sampling the **k**-space acquisition is not the common trend in acquisition. Nowadays, due to time restrictions, most acquisitions are usually accelerated by using parallel MRI (pMRI) reconstruction techniques, which allow to increase the acquisition rate via subsampled acquisitions of the **k**-space. This acceleration goes together with an artifact known as *aliasing*.

Many reconstruction methods have been proposed in order to suppress the aliasing created by this subsampling, with SENSE (Sensitivity Encoding for Fast MRI) [12] and GRAPPA (Generalized Autocalibrating Partially Parallel Acquisition) [13] citegrappa being dominant among them. From a statistical point of view, both reconstruction methods will affect the stationarity of the noise in the reconstructed data, i.e. the spatial distribution of the noise across the image. As a result, if SENSE is used, the magnitude signal may be considered Rician distributed [14,15] but the value of the statistical parameters and, in particular, the variance of noise σ_n^2, will vary for different image locations, i.e. it becomes **x**-dependent. Similarly, if GRAPPA is used, the CMS may be approximated by a non-stationary nc-χ distribution [15,16] with effective parameters.

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Noise estimators proposed in literature are based on the assumption of a single σ_n^2 value for all the pixels in the image, assuming either a Rician model [17,18,5,4,19,20] or an nc-χ [9,19,21,10]. Accordingly, those methods do not apply when dealing with pMRI and non-stationary noise. Noise estimators must therefore be reformulated in order to cope with these new image modalities.

In this paper we propose different methodologies to estimate the spatially distributed variance of noise σ_n^2 from the magnitude signal when SENSE or GRAPPA are used as pMRI technique.

2. Noise statistical models in pMRI

As previously stated, most noise estimation methods in literature rely on the assumption of a single value of σ_n^2 for every pixel within the image. However, this is no longer the case when pMRI protocols are considered.

In multiple coil systems, the acquisition rate may be increased by subsampling the **k**-space data [22,23], while reducing phase distortions when strong magnetic field gradients are present. The immediate effect of the **k**-space subsampling is the appearance of aliased replicas in the image domain retrieved at each coil. In order to suppress or correct this aliasing, pMRI combines the redundant information from several coils to reconstruct a single non-aliased image domain.

The commonly used (stationary) Rician and nc-χ models do not necessarily hold in this case. Depending on the way the information from each coil is combined, the statistics of the image will follow different distributions. It is therefore necessary to study the behavior of the data for a particular reconstruction method. We will focus on two of the most popular methods, SENSE [12] and GRAPPA [13], in their most basic formulation.

In the following sections we will assume an L-coil configuration, with L being the number of coils in the system. $s_l^S(\mathbf{k})$ is the subsampled signal at the l-th coil of the **k**-space ($l = 1, \cdots, L$), $S_l^S(\mathbf{x})$ is the subsampled signal in the image domain, i.e., the **x**-space, and r is the subsampling rate. The **k**-space data at each coil can be accurately described by an Additive White Gaussian Noise (AWGN) process, with zero mean and variance σ_K^2:

$$s_l^S(\mathbf{k}) = a_l(\mathbf{k}) + n_l\left(\mathbf{k}; \sigma_{K_l}^2\right), \quad l = 1, \cdots, L \tag{1}$$

with $a_l(\mathbf{k})$ the noise-free signal and $n_l\left(\mathbf{k}; \sigma_{K_l}^2\right) = n_{l_r}\left(\mathbf{k}; \sigma_{K_l}^2\right) + j \cdot n_{l_i}\left(\mathbf{k}; \sigma_{K_l}^2\right)$ the AWGN process, which is initially assumed stationary so that $\sigma_{K_l}^2$ does not depend on **k**.

The complex **x**-space is obtained as the inverse Discrete Fourier Transform (iDFT) of $s_l^S(\mathbf{k})$ for each slice or volume, so the noise in the complex **x**-space is still Gaussian [15]:

$$S_l^S(\mathbf{x}) = A_l(\mathbf{x}) + N_l\left(\mathbf{x}; \sigma_l^2\right), \quad l = 1, \cdots, L$$

where $N_l(\mathbf{x}; \sigma_l^2) = N_{l_r}(\mathbf{x}; \sigma_l^2) + jN_{l_i}(\mathbf{x}; \sigma_l^2)$ is also a complex AWGN process (note we are assuming that there are not any spatial correlations) with zero mean and covariance matrix:

$$\boldsymbol{\Sigma} = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1L} \\ \sigma_{21} & \sigma_2^2 & \cdots & \sigma_{2L} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{L1} & \sigma_{L2} & \cdots & \sigma_L^2 \end{pmatrix}. \tag{2}$$

The relation between the noise variances in the **k**- and **x**-domains is given by the number of points used for the iDFT:

$$\sigma_l^2 = \frac{r}{|\Omega|}\sigma_{K_l}^2,$$

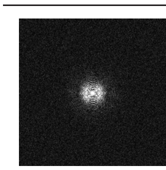
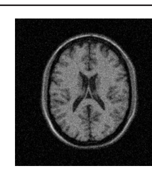
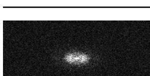
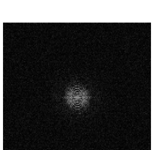
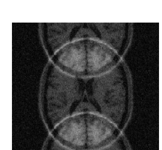
with $|\Omega|$ the final number of pixels in the field of view (FOV). Note that the final noise power is greater than in the fully sampled case due to the reduced **k**-space averaging, as it will be the case with SENSE (see below). On the contrary, the iDFT may be computed after zero-padding the missing (not sampled) **k**-space lines, and then we have [16]:

$$\sigma_l^2 = \frac{1}{|\Omega| \cdot r}\sigma_{K_l}^2.$$

In the latter case the noise power is reduced with respect to the fully sampled case, since we average exactly the same number of samples but only 1 of each r of them contributes a noise sample (this will also be the case with GRAPPA). Finally, note that although the level of noise is smaller in GRAPPA due to the zero padding, the SNR does not increase, since the zero padding produces also a reduction of the level of the signal.

Relations between the variance of noise in complex **x**-space and **k**-space for each coil are summarized in Table 1.

Table 1
Relations between the variance of noise in complex MR data for each coil in the **k**-space and the image domain.

Noise relations			
k-space	Parameters	**x**-space	Relation
	Fully sampled, $\sigma_{K_l}^2$ **k**-size: $\lvert\Omega\rvert$		$\sigma_l^2 = \frac{1}{\lvert\Omega\rvert}\sigma_{K_l}^2$, **x**-size: $\lvert\Omega\rvert$
	Subsampled r, $\sigma_{K_l}^2$ **k**-size: $\lvert\Omega\rvert/r$		$\sigma_l^2 = \frac{r}{\lvert\Omega\rvert}\sigma_{K_l}^2$, **x**-size: $\lvert\Omega\rvert/r$ (SENSE)
	Subsampled r, $\sigma_{K_l}^2$ **k**-size: $\lvert\Omega\rvert$ (zero padded)		$\sigma_l^2 = \frac{1}{\lvert\Omega\rvert \cdot r}\sigma_{K_l}^2$, **x**-size: $\lvert\Omega\rvert$ (GRAPPA)