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Advanced theory of driven birdcage resonator with losses for biomedical magnetic resonance imaging and spectroscopy

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Abstract

A complete time-dependent physics theory of symmetric unperturbed driven hybrid birdcage resonator was developed for general application. In particular, the theory can be applied for radiofrequency (RF) coil engineering, computer simulations of coil-sample interaction, etc. Explicit time dependence is evaluated for different forms of driving voltage. The major steps of the solution development are shown and appropriate explanations are given. Green's functions and spectral density formula were developed for any form of periodic driving voltage. The concept of distributed power losses based on transmission line theory is developed for evaluation of local losses of a coil. Three major types of power losses are estimated as equivalent series resistances in the circuit of the birdcage resonator. Values of generated resistances in legs and end-rings are estimated. An application of the theory is shown for many practical cases. Experimental curve of B_1 field polarization dependence is measured for eight-sections birdcage coil. It was shown that the steady-state driven resonance frequencies do not depend on damping factor unlike the free oscillation (transient) frequencies. An equivalent active resistance is generated due to interaction of RF electromagnetic field with a sample. Resistance of the conductor (enhanced by skin effect), Eddy currents and dielectric losses are the major types of losses which contribute to the values of generated resistances. A biomedical sample for magnetic resonance imaging and spectroscopy is the source of the both Eddy current and dielectric losses of a coil. As demonstrated by the theory, Eddy current loss is the major effect of coil shielding.
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1. Introduction

The birdcage-type resonator has been introduced in [1] as a highly homogeneous radiofrequency (RF) coil for whole-body imaging.

Two main approaches to the theoretical description of birdcage resonator physics system have been published. One approach considers the system as a complicated oscillator [2–5]; another represents the birdcage as a transmission line [6–9].

The term *hybrid birdcage* resonator implies a more complicated form of the birdcage coil which has both low- and high-pass features [4,10,11]; however, there are other types of birdcage resonators such as double-tuned coils [12,13], etc.

The real and the imaginary parts of an impedance may be referred to as active and reactive resistances respectively. Here a hybrid-type birdcage coil with active resistance elements in its electronic circuit (Fig. 1) will be discussed to describe a complete physics theory of a symmetric unperturbed birdcage resonator.

Many previous publications [3,4] discuss the practical forms of birdcage resonators like low- and high-pass, which do not contain active resistance elements in their circuits. However, an active resistance is always present in real RF coils at room temperature. Any type of resonator loaded with a sample for experimental nuclear magnetic resonance (NMR) measurements due to inductive and capacitive losses has an equivalent resistance, and acts as an active series resistance element in an electronic circuit [14,15]. Hence, the loaded quality factor (Q) decreases essentially [16]. Also, there is a useful power transduction for nuclear spins excitation, which induces an equivalent resistance as well. The presence of an active resistance in any of above

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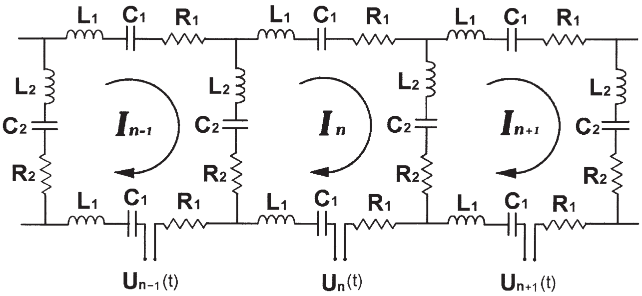

Fig. 1. Three consecutive meshes of driven hybrid birdcage ladder network. All meshes of the network are driven by different periodic voltage sources $U_n(t)$ with low internal impedances.

mentioned forms is a necessary condition of power transduction from a RF source to a resonator. Hence, it is appropriate to consider theory of a resonator with active resistance elements included in its circuit. Previously, there was a brief theoretical discussion of symmetric unperturbed driven birdcage resonator with losses [2], though it is incomplete and mathematically inaccurate.

The physics system of the birdcage resonator represents a hybrid between continuous and discrete systems, since it has both continuous and discrete variables. That is the reason why the solution of electronic motion equations can not be obtained straightforwardly by standard methods [17,18]. Evaluation of the solution involves mathematical methods such as Theory of Groups Representations, Vector Analysis, Matrices and Determinants, Differential Equations, among others.

2. Theoretical solution development

2.1. Setting equations of electronic motion

According to the Kirchhoff rule, the voltage balance of the nth loop (Fig. 1) can be expressed by the equation:

$$\begin{aligned}2L_1\frac{d}{dt}I_n + 2R_1I_n + \frac{2}{C_1}\int I_n dt + L_2\frac{d}{dt}(I_n - I_{n-1}) \\ + R_2(I_n - I_{n-1}) + \frac{1}{C_2}\int (I_n - I_{n-1})dt + L_2\frac{d}{dt}(I_n - I_{n+1}) \\ + R_2(I_n - I_{n+1}) + \frac{1}{C_2}\int (I_n - I_{n+1})dt = U_n(t) \qquad (1)\end{aligned}$$

To eliminate integral signs, it is reasonable to apply differential operation to both parts of the equation:

$$\begin{aligned}2L_1\frac{d^2}{dt^2}I_n + 2R_1\frac{d}{dt}I_n + \frac{2}{C_1}I_n + L_2\frac{d^2}{dt^2}(I_n - I_{n-1}) \\ + R_2\frac{d}{dt}(I_n - I_{n-1}) + \frac{1}{C_2}(I_n - I_{n-1}) + L_2\frac{d^2}{dt^2}(I_n - I_{n+1}) \\ + R_2\frac{d}{dt}(I_n - I_{n+1}) + \frac{1}{C_2}(I_n - I_{n+1}) = \frac{d}{dt}U_n(t) \qquad (2)\end{aligned}$$

For convenience, assume time dependence: $I_n(t) \sim e^{\lambda t}$. Hence, the equation may be represented in the form:

$$\begin{aligned}2\left[\lambda^2(L_1 + L_2) + \lambda(R_1 + R_2) + \left(\frac{1}{C_1} + \frac{1}{C_2}\right)\right] \\ \times I_n - \left[\lambda^2 L_2 + \lambda R_2 + \frac{1}{C_2}\right](I_{n-1} + I_{n+1}) = \frac{d}{dt}U_n(t) \quad (3)\end{aligned}$$

which is the same as:

$$aI(t,n) + b[I(t,n-1) + I(t,n+1)] = F(t,n) \qquad (4)$$

where elements of the equation are:

$$a = 2\left[\lambda^2(L_1 + L_2) + \lambda(R_1 + R_2) + \left(\frac{1}{C_1} + \frac{1}{C_2}\right)\right] \qquad (5)$$

$$b = -\lambda^2 L_2 - \lambda R_2 - \frac{1}{C_2} \qquad (6)$$

$$I(t,n) = I_n \qquad (7)$$

$$I(t,n-1) = I_{n-1} \qquad (8)$$

$$I(t,n+1) = I_{n+1} \qquad (9)$$

$$F(t,n) = \frac{d}{dt}U_n(t) \qquad (10)$$

Note, the whole system of Eq. (4) may be represented as:

$$\hat{A}|I(t)\rangle = |F(t)\rangle \qquad (11)$$

where $\hat{A}$ is a matrix operator and $|I(t)\rangle$, $|F(t)\rangle$ are column vectors:

$$\hat{A} = \begin{vmatrix} a & b & 0 & . & 0 & b \\ b & a & b & . & 0 & 0 \\ 0 & b & a & . & 0 & 0 \\ . & . & . & . & . & . \\ 0 & 0 & 0 & . & a & b \\ b & 0 & 0 & . & b & a \end{vmatrix} \quad |I(t)\rangle = \begin{vmatrix} I(t,1) \\ I(t,2) \\ . \\ . \\ . \\ I(t,N) \end{vmatrix} \quad |F(t)\rangle = \begin{vmatrix} F(t,1) \\ F(t,2) \\ . \\ . \\ . \\ F(t,N) \end{vmatrix} \qquad (12)$$

The next step of the solution development is an evaluation of the matrix operator, $\hat{A}$. To evaluate an operator it is appropriate to find its eigenvalues and eigenfunctions.

2.2. Finding eigenvalues and eigenvectors of the space operator

Eigenvalues A_n of the matrix operator $\hat{A}$ must satisfy the equation:

$$\hat{A}|X_n\rangle = |X_n\rangle A_n \qquad (13)$$

where $|X_n\rangle$ are the eigenvectors of the operator.

To find eigenvalues and eigenvectors of the matrix operator $\hat{A}$, it is appropriate to know the properties of a