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COmplex-Model-Based Estimation of thermal noise for fMRI data in the presence of artifacts

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Abstract

Due to the presence of artifacts induced by fast-imaging acquisition in functional magnetic resonance imaging (fMRI) studies, it is very difficult to estimate the variance of thermal noise by traditional methods in magnitude images. Moreover, the existence of incidental phase fluctuations impairs the validity of currently available solutions based on complex datasets. In this article, a time-domain model is proposed to generalize the analysis of complex datasets for nonbrain regions by incorporating artifacts and phase fluctuations. Based on this model, a novel estimation schema has been developed to find an appropriate set of voxels in nonbrain regions according to their levels of artifact and phase fluctuation. In addition, noise intensity from these voxels is estimated. The whole schema is named COmplex-Model-Based Estimation (COMBE). Theoretical and experimental results demonstrate that the proposed COMBE method provides a better estimation of thermal noise in fMRI studies compared with previously proposed methods and suggest that the new method can adapt to a broader range of applications, such as functional connectivity studies, evaluation of sequence designs and reconstruction schemas.

Keywords: Thermal noise; Complex-valued model; COmplex-Model-Based Estimation; fMRI; Human brain

1. Introduction

Thermal noise in magnetic resonance (MR) images is a very important parameter. The estimation of thermal noise not only provides measurements of the quality of a magnetic resonance imaging (MRI) system [1] and quantification of an MR signal, especially signal-to-noise ratio (SNR) for functional MRI (fMRI) signal [2], but also offers a general measure to evaluate the performance of MRI sequences [3] and reconstruction schemas [4].

Analytical estimation methods that determine thermal noise have been extensively studied. In most cases, thermal noise is determined from magnitude images, which can be modeled as a Rician distribution that has no analytical solution. When the SNR is high, the Rician distribution can be approximated as Gaussian in nature, and thermal noise can be estimated as the standard deviation of magnitude [5]. When the signal is zero, the Rician model evolves to a Rayleigh distribution, and thermal noise can be estimated by dividing the standard deviation of magnitude with a correctional factor (about 0.655) [6,7].

Technological development and clinical research applications of fMRI methods have generated three new challenges in estimating noise. First, unlike anatomical images, fMRI datasets in high SNR regions, such as the brain, contain significant temporal signal changes, hence invalidating the Gaussian method. Temporal signal changes may include fluctuations in blood-oxygenation-level-dependent (BOLD) signals induced by tasks or physiologic noise during rest [8]. Second, the presence of significant artifacts in background regions (nonbrain regions), acquired by fast-imaging methods such as Echo Planar Imaging (EPI) [9] and spiral [10] pulse sequences, makes the Rayleigh method inapplicable. To solve this problem, researchers have developed several methods, including averaging variances over real and imaginary channels (Average method) [11–13], maximum likelihood (ML)-based estimations [12,14] and a double-acquisition method employing the analytical form of even moments of the Rician distribution [15].

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The final problem seen in fMRI datasets is incidental phase fluctuations, which are derived from various time-dependent sources of variation, including flip-angle inhomogeneity, filter responses, system delay, noncentered sampling windows and others [16]. However, the above-mentioned methods did not explicitly take phase fluctuation into account as a random variable.

In this article, a new method is proposed for fMRI datasets. It estimates thermal noise in the presence of image artifacts and phase fluctuations for fMRI datasets. By analyzing the real and imaginary channels of complex-valued data in the time domain, it becomes evident that thermal noise can be accurately estimated. Theoretical simulations and experimental results demonstrate that the new method, compared with previously proposed methods, provides a better estimation of thermal noise and a higher capacity for a broader range of artifact-to-noise ratios (ANRs) and phase fluctuations. Thus, the new method is suitable for various applications, including functional connectivity studies, sequence evaluation and reconstruction evaluation.

2. Theory

It is well known that in the brain, real [$R_b(t)$] and imaginary [$I_b(t)$] channels in a given voxel of reconstructed fMRI datasets have three components. These components are: the magnitude of the signal $S(t)$, the phase of the signal $\theta(t)$ and the thermal noise $n(t)$ at time t. The time-domain model within a given voxel can be expressed as:

$$R_b(t) = S(t)\cos(\theta(t)) + n_1(t)$$

$$I_b(t) = S(t)\sin(\theta(t)) + n_2(t) \quad (1)$$

where $n_1(t)$ and $n_2(t)$ are additive thermal measurement noise [17,18]. As previously described, the magnitude and phase portions of the signal are temporally varying quantities. Thus, they may be modeled by temporally constant mean-varying and time-varying portions $S(t)=S+\Delta S(t)$ and $\theta(t)=\theta+\Delta\theta(t)$. The temporally constant means of magnitude and phase are S and θ, while their time-varying portions are $\Delta S(t)$ and $\Delta\theta(t)$, respectively.

Any signal that is present in the background region of reconstructed fMRI datasets is due to ghosting artifacts. Thus, it is described by a decreased version of the original signal. The original magnitude signal $S(t)$ in Eq. (1) is decreased by an artifact proportionality factor γ to yield $\gamma S(t)=\gamma S+\gamma\Delta S(t)$ instead of $S(t)$ for an artifact-to-noise model.

In fMRI data, the mean magnitude signal is usually much larger than its temporal variation $S \gg \Delta S(t)$, so that the artifact magnitude signal is also much larger than its temporal variation $\gamma S \gg \gamma\Delta S(t)$. This allows the variation of the artifact signal to be neglected when $\gamma S(t)$ is comparable to thermal noise. Thus, the artifact $\gamma S(t)$ can be taken as a temporally constant quantity a, which is called the artifact level. The artifact-to-noise complex model can be written as:

$$R(t) = a\cos(\theta + \Delta\theta(t)) + n_1(t)$$

$$I(t) = a\sin(\theta + \Delta\theta(t)) + n_2(t). \quad (2)$$

In nonartifact voxels, $a=0$, so that real and imaginary channels consist only of noise. The phase fluctuation $\Delta\theta(t)$ in fMRI data is relatively small. This allows Eq. (2) to be written as:

$$R(t) = a\cos(\theta(t)) + n_1(t) \approx a(\cos\theta - \sin\theta \cdot \Delta\theta(t)) + n_1(t)$$

$$I(t) = a\sin(\theta(t)) + n_2(t) \approx a(\sin\theta + \cos\theta \cdot \Delta\theta(t)) + n_2(t) \quad (3)$$

with the use of trigonometric addition formulas for sines and cosines along with small-angle approximations.

By definition, thermal noise $n_1(t)$ and $n_2(t)$ in the two channels are mutually independent and identically distributed with zero means and variances σ_0^2. Additionally, the mean and the variance of phase fluctuation are zero and σ_θ^2, respectively. With the above model specifications, the mean (expected) values for the real and imaginary parts of the artifact-to-noise complex model are:

$$\mu_R = a\cos\theta$$

$$\mu_I = a\sin\theta \quad (4)$$

while their variances are:

$$\sigma_R^2 = a^2\sin^2\theta \cdot \sigma_\theta^2 + \sigma_0^2$$

$$\sigma_I^2 = a^2\cos^2\theta \cdot \sigma_\theta^2 + \sigma_0^2. \quad (5)$$

Since we do not assume any particular distribution for thermal noise and phase fluctuations, we do not have a likelihood and cannot estimate model parameters with maximum likelihood estimates (MLEs). Instead, we will estimate model parameters with method of moment estimators (MMEs). MMEs are found by equating population moments to sample moments [19]. MMEs for the artifact level a and the mean phase θ are found by equating population means to sample means (first moments). This yields the equations:

$$\overline{R} = \hat{a}\cos(\hat{\theta}); \quad \overline{I} = \hat{a}\sin(\hat{\theta}) \quad (6)$$

where $\bar{R}$ and $\bar{I}$ are the sample means of real and imaginary channels. The solution to these two equations with two unknowns yields MMEs that are:

$$\hat{a} = \sqrt{(\overline{R})^2 + (\overline{I})^2}; \quad \hat{\theta} = \tan^{-1}(\overline{I}/\overline{R}). \quad (7)$$

For convenience, $\hat{a}$ is named the artifact level, while $\hat{\theta}$ is the estimated phase mean. MMEs for variances σ_0^2 and σ_θ^2