Physica B 499 (2016) 49–56

Contents lists available at ScienceDirect

Physica B

journal homepage: www.elsevier.com/locate/physb

A nonlinear lattice model for Heisenberg helimagnet and spin wave instabilities

A. Ludvin Felcy [a], M.M. Latha [b,*], C. Christal Vasanthi [b]

[a] Department of Physics, St. Xavier's Catholic College of Engineering, Chunkankadai 629003, India
[b] Department of Physics, Women's Christian College, Nagercoil 629001, India

ARTICLE INFO

Article history:
Received 22 December 2015
Received in revised form
2 July 2016
Accepted 4 July 2016
Available online 5 July 2016

Keywords:
Heisenberg helimagnet
Soliton
Modulational Instability

ABSTRACT

We study the dynamics of a Heisenberg helimagnet by presenting a square lattice model and proposing the Hamiltonian associated with it. The corresponding equation of motion is constructed after averaging the Hamiltonian using a suitable wavefunction. The stability of the spin wave is discussed by means of Modulational Instability (MI) analysis. The influence of various types of inhomogeneities in the lattice is also investigated by improving the model.

1. Introduction

Helimagnetism is an incommensurate form of magnetic ordering that results from the competition between ferromagnetic and antiferromagnetic exchange interactions and is typically only observed at liquid helium temperatures [1,2]. In helimagnets, spins of neighbouring magnetic moments arrange themselves in a spiral or pattern with a characteristic turn angle of somewhere between 0° and 180°. Rare earth metals like terbium (Tb), dysprosium (Dy) and holmium (Ho) remain in a helical phase above a particular temperature and at low temperatures. Many experimental and theoretical studies have been carried out on helimagnetic systems by proposing different models [3–6]. Masuda et al. [3] measured the dispersion of spin wave excitations in the vicinity of magnetic Bragg reflection and hence calculated the exchange constants in $LiCu_2O_2$ helimagnet. Chandra and Coleman [6] introduced a fictitious twist vector potential in the Heisenberg Hamiltonian and showed that large quantum fluctuations induce an anisotropy in Heisenberg spiral structure. Recently Daniel and Beula [7] found soliton spin excitations in a Heisenberg helimagnet in semi-classical limit by considering a model in which the helical spin arrangement in the magnetic system is introduced in analogy with the twist deformation in a cholesteric liquid crystal. Experimentally Drillat et al. [8] measured the turn angle and have shown that it decreases with decreasing temperature due to enhancement of ferromagnetic interaction. A number of experimental models and theoretical studies have been carried out recently on helimagnet. Recent progress on synthesis of new class of helimagnetic structures revives the interest to the case of chiral helimagnet with an easy-plane anisotropy [9]. Recently, a 2D view in helimagnet $LiCu_2O_2$ has been confirmed from its renormalized classical behaviour of short range in plane magnetic correlation length based on resonant soft X-ray scattering measurements [10–14]. But there is no analytical study yet reported on the nonlinear dynamics of the lattice model of the helimagnetic system. Inspired by this, in this paper, we propose a Hamiltonian for a lattice model of helimagnetic system and investigate the solitonic aspects by constructing the equations of motion after averaging the Hamiltonian using a suitable wave function. The plan of the paper is as follows: In Section 2, we propose a model Hamiltonian including bilinear exchange and anisotropic interactions for a homogeneous helimagnet. After deriving the dynamical equation of motion in the discrete form, we construct the soliton solution by making a continuum approximation. In Section 3, the resulting nonlinear equation in discrete form is analysed using Modulational Instability analysis and the effect of interaction parameters on soliton stability is investigated. In Sections 4 and 5, we investigate the nonlinear dynamics of inhomogeneous Heisenberg helimagnetic spin chain by taking into consideration the stability aspects of soliton under various types of inhomogeneities. The results are concluded in Section 6.

* Corresponding author.
E-mail address: lathaisaac@yahoo.com (M.M. Latha).

http://dx.doi.org/10.1016/j.physb.2016.07.005
0921-4526/

2. The model

2.1. Homogeneous case

The Hamiltonian for our system (Fig. 1) consists of seven parts and it is written as

$$\tilde{H} = -\sum_{i,j}\Big[\tilde{J}_1(\vec{S}_{i,j}\cdot\vec{S}_{i+1,j}) + \tilde{J}_2(\vec{S}_{i,j}\cdot\vec{S}_{i,j+1}) + \tilde{J}_3(\vec{S}_{i,j}\cdot\vec{S}_{i+1,j+1}) + \tilde{\Gamma}_1[\hat{k}\cdot(\vec{S}_{i,j}\times\vec{S}_{i+1,j})]^2 + \tilde{\Gamma}_2[\hat{k}\cdot(\vec{S}_{i,j}\times\vec{S}_{i,j+1})]^2 + \tilde{\Gamma}_3[\hat{k}\cdot(\vec{S}_{i,j}\times\vec{S}_{i+1,j+1})]^2 - \tilde{A}(\vec{S}^{z}_{i,j})^2\Big], \tag{1}$$

where $\tilde{J}_1, \tilde{J}_2, \tilde{J}_3$, represent the constant coefficients of bilinear exchange interaction along the X, Y and the diagonal directions respectively. Γ_1, Γ_2, Γ_3 stand for helical arrangement of spins in the magnetic system along the X, Y and the diagonal directions respectively and $\tilde{A}$ represents the crystal field anisotropic interaction. $\vec{S}_{i,j} = (S^x_{i,j}, S^y_{i,j}, S^z_{i,j})$ denotes the spin at the lattice site. Now, we express the spin Hamiltonian (1) in the dimensionless form by introducing the dimensionless spin $\hat{S}_{i,j} = \frac{\vec{S}_{i,j}}{\hbar}$ and by defining $\hat{S}^{\pm}_{i,j} = \hat{S}^{x}_{i,j} \pm i\hat{S}^{y}_{i,j}$. The Hamiltonian now becomes

$$H = -\sum_{i,j}\Big[\frac{J_1}{2S^2}(\hat{S}^{+}_{i,j}\hat{S}^{-}_{i+1,j} + \hat{S}^{-}_{i,j}\hat{S}^{+}_{i+1,j} + 2\hat{S}^{z}_{i,j}\hat{S}^{z}_{i+1,j}) + \frac{J_2}{2S^2}(\hat{S}^{+}_{i,j}\hat{S}^{-}_{i,j+1} + \hat{S}^{-}_{i,j}\hat{S}^{+}_{i,j+1} + 2\hat{S}^{z}_{i,j}\hat{S}^{z}_{i,j+1}) + \frac{J_3}{2S^2}(\hat{S}^{+}_{i,j}\hat{S}^{-}_{i+1,j+1} + \hat{S}^{-}_{i,j}\hat{S}^{+}_{i+1,j+1} + 2\hat{S}^{z}_{i,j}\hat{S}^{z}_{i+1,j+1}) - \frac{\Gamma_1}{4S^4}(\hat{S}^{+}_{i,j}\hat{S}^{-}_{i+1,j}\hat{S}^{+}_{i,j}\hat{S}^{-}_{i+1,j} - 2\hat{S}^{+}_{i,j}\hat{S}^{+}_{i+1,j}\hat{S}^{-}_{i,j}\hat{S}^{-}_{i+1,j} + \hat{S}^{-}_{i,j}\hat{S}^{+}_{i+1,j}\hat{S}^{-}_{i,j}\hat{S}^{+}_{i+1,j}) - \frac{\Gamma_2}{4S^4}(\hat{S}^{+}_{i,j}\hat{S}^{-}_{i,j+1}\hat{S}^{+}_{i,j}\hat{S}^{-}_{i,j+1} - 2\hat{S}^{+}_{i,j}\hat{S}^{+}_{i,j+1}\hat{S}^{-}_{i,j}\times\hat{S}^{-}_{i,j+1} + \hat{S}^{-}_{i,j}\hat{S}^{+}_{i,j+1}\hat{S}^{-}_{i,j}\hat{S}^{+}_{i,j+1}) - \frac{\Gamma_3}{4S^4}(\hat{S}^{+}_{i,j}\hat{S}^{-}_{i+1,j+1}\hat{S}^{+}_{i,j}\hat{S}^{-}_{i+1,j+1} - 2\hat{S}^{+}_{i,j}\hat{S}^{+}_{i+1,j+1}\hat{S}^{-}_{i,j}\hat{S}^{-}_{i+1,j+1} + \hat{S}^{-}_{i,j}\hat{S}^{+}_{i+1,j+1}\hat{S}^{-}_{i,j}\hat{S}^{+}_{i+1,j+1}) - \frac{A}{S^2}(\hat{S}^{z}_{i,j})^2\Big]. \tag{2}$$

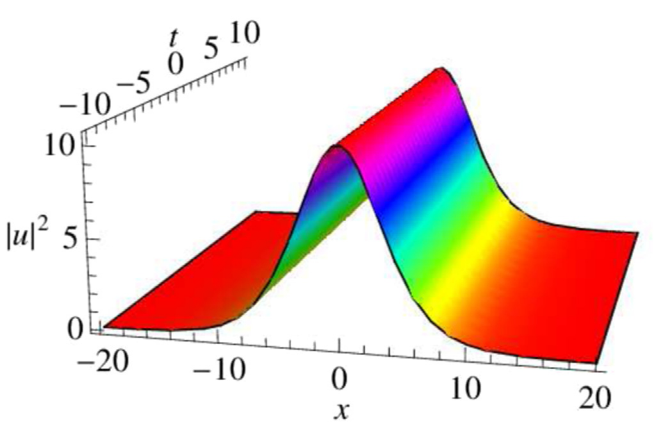

Fig. 1. Solitonic profile for c=0.002, $J_1 = 12$, $J_2 = 6$, $J_3 = 3$, $A = -0.1$, $\gamma = 0.6$, y=0.2, $\Gamma_1 = 0.06$, $\Gamma_2 = -0.9$ and $\Gamma_3 = 0.5$.

In Eq. (2), $H = \frac{\tilde{H}}{\hbar^2 S^2}$; $J_1 = \tilde{J}_1$; $J_2 = \tilde{J}_2$; $J_3 = \tilde{J}_3$ and $A = \tilde{A}$. Bosonizing the Hamiltonian using the HP representation [15] of spin operators given by $\hat{S}^{+}_{i,j} = (2S)^{\frac{1}{2}}[1 - \frac{a^{\dagger}_{i,j}a_{i,j}}{2S}]^{\frac{1}{2}}a_{i,j}$, $\hat{S}^{-}_{i,j} = (2S)^{\frac{1}{2}}a^{\dagger}_{i,j}[1 - \frac{a^{\dagger}_{i,j}a_{i,j}}{2S}]^{\frac{1}{2}}$ and $\hat{S}^{z}_{i,j} = [S - a^{\dagger}_{i,j}a_{i,j}]$ and using the commutation relations $[a_{i,j}, a^{\dagger}_{i,j}] = 1$, $[a_{i,j}, a_{i,j}] = 0$ and the expressions

$$\frac{\hat{S}^{+}_{i,j}}{S} = \sqrt{2}\left[1 - \frac{\epsilon^2}{4}a^{\dagger}_{i,j}a_{i,j} - O(\epsilon^4)\right]\epsilon a_{i,j}, \tag{3}$$

$$\frac{\hat{S}^{-}_{i,j}}{S} = \sqrt{2}\,\epsilon a^{\dagger}_{i,j}\left[1 - \frac{\epsilon^2}{4}a^{\dagger}_{i,j}a_{i,j} - O(\epsilon^4)\right], \tag{4}$$

Eq. (2) can be written as a power series in ϵ as

$$H = \sum_{i,j}\Big\{(A - J_1 - J_2 - J_3) + \epsilon^2[(J_1 + J_2 + J_3 - 2A)a_{i,j}a^{\dagger}_{i,j} + J_1(a_{i+1,j}a^{\dagger}_{i,j} - a_{i,j}a^{\dagger}_{i+1,j} - a_{i+1,j}a^{\dagger}_{i+1,j}) + J_2(a_{i,j+1}a^{\dagger}_{i,j} - a_{i,j}a^{\dagger}_{i,j+1} - a_{i,j+1}a^{\dagger}_{i,j+1}) + J_3(a_{i+1,j+1}a^{\dagger}_{i,j} + a_{i,j}a^{\dagger}_{i+1,j+1} - a_{i+1,j+1}a^{\dagger}_{i+1,j+1})] + \frac{\epsilon^4}{4}[4Aa^{2}_{i,j}a^{\dagger 2}_{i,j} + J_1(a_{i,j}a_{i+1,j}a^{\dagger 2}_{i,j} + a^{\dagger 2}_{i,j}a^{\dagger}_{i,j}a^{\dagger}_{i+1,j} - 4a_{i,j}a_{i+1,j}a^{\dagger}_{i,j}\times a^{\dagger}_{i+1,j} + a^{\dagger 2}_{i+1,j}a^{\dagger}_{i,j}a^{\dagger}_{i+1,j} + a_{i,j} + a_{i+1,j}a^{\dagger 2}_{i+1,j}) + J_2(a_{i,j}a_{i,j+1}a^{\dagger 2}_{i,j} + a^{\dagger 2}_{i,j}a^{\dagger}_{i,j}a^{\dagger}_{i,j+1} - 4a_{i,j}a_{i,j+1}a^{\dagger}_{i,j}a^{\dagger}_{i,j+1} + a^{\dagger 2}_{i,j+1}a^{\dagger}_{i,j}a^{\dagger}_{i,j+1} + a_{i,j} + a_{i,j+1}\times a^{\dagger 2}_{i,j+1}) + J_3(a_{i,j}a_{i+1,j+1}a^{\dagger 2}_{i,j} + a^{\dagger 2}_{i,j}a^{\dagger}_{i,j}a^{\dagger}_{i+1,j+1} - 4a_{i,j}a_{i+1,j+1}a^{\dagger}_{i,j}\times a^{\dagger}_{i+1,j+1} + a^{\dagger 2}_{i+1,j+1}a^{\dagger}_{i,j}a^{\dagger}_{i+1,j+1} + a_{i,j}a_{i+1,j+1}a^{\dagger 2}_{i+1,j+1}) + \frac{\Gamma_1}{4}(a^{\dagger}_{i+1,j}\times a^{\dagger 2}_{i,j} - 2a_{i,j}a_{i+1,j}a^{\dagger}_{i,j}a^{\dagger}_{i+1,j} + a^{\dagger 2}_{i,j}a^{\dagger 2}_{i+1,j}) + \frac{\Gamma_2}{4}(a^{\dagger}_{i,j+1}a^{\dagger 2}_{i,j} - 2a_{i,j}a_{i,j+1}\times a^{\dagger}_{i,j}a^{\dagger}_{i,j+1} + a^{\dagger 2}_{i,j}a^{\dagger 2}_{i,j+1}) + \frac{\Gamma_3}{4}(a^{\dagger}_{i+1,j+1}a^{\dagger 2}_{i,j} - 2a_{i,j}a_{i+1,j+1}a^{\dagger}_{i,j}a^{\dagger}_{i+1,j+1} + a^{\dagger 2}_{i,j}a^{\dagger 2}_{i+1,j+1})\Big\}. \tag{5}$$

Next we construct the equation of motion for the boson operator using

$$i\hbar\frac{\partial a_n}{\partial t} = [a_n, H] = F(a^{\dagger}_n, a_n, a^{\dagger}_{n+1}, a_{n+1}). \tag{6}$$

Introducing Glauber's coherent-state representation [16] for the bosonic operators, $a^{\dagger}_{i,j}|u\rangle = u^{*}_{i,j}|u\rangle$, $a_{i,j}|u\rangle = u_{i,j}|u\rangle$, with $\langle u|u\rangle = 1$, where $u_{i,j}$ is the coherent amplitude of the operator $a_{i,j}$ for the system in the state $|u\rangle$, we write down the equation of motion using Eq. (6) for the average $\langle u|a_{i,j}|u\rangle$ as