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Long-time averaged dynamics of a Bose–Einstein condensate in a bichromatic optical lattice with external harmonic confinement

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ABSTRACT

The dynamics of a Bose–Einstein condensate are examined numerically in the presence of a one-dimensional bichromatic optical lattice (BCOL) with external harmonic confinement in the strongly interacting regime. The condensate is excited by a focusing stirring red laser. Two realizations of the BCOL are considered, one with a rational and the other with an irrational ratio of the two constituting wave lengths. The system is simulated by the time-dependent Gross Pitaevskii equation that is solved using the Crank Nicolson method in real time. It is found that for a weak BCOL, the long-time averaged physical observables of the condensate respond only very weakly (or not at all) to changes in the secondary OL depth V_1 showing that under these conditions the harmonic trap plays a dominant role in governing the dynamics. However, for a much larger strength of the BCOL, the response is stronger as it begins to compete with the external harmonic trap, such that the frequency of Bloch oscillations of the bosons rises with V_1 yielding higher time-averages. Qualitatively there is no difference between the dynamics of the condensate resulting from the use of a rational or irrational ratio of the wavelengths since the external harmonic trap washes it out. It is further found that in the presence of an external harmonic trap, the BCOL acts in favor of superflow.

1. Introduction

There has been considerable interest in the study of Bose–Einstein condensate (BEC) dynamics in a bichromatic optical lattice (BCOL) [1–9] because it offers a large flexibility that is realized by the tuning of a few experimental parameters. One important property is that it can be tuned to introduce a quasidisordered environment in which the properties of the bosons are largely determined by the strength of BCOL disorder. The realization of this quasidisorder has hitherto been achieved by the superposition of laser beams with an irrational ratio of their wavelengths [10–12]. Therefore, the resulting disorder and its effects thereof on the properties of a BEC are of significant interest [6]. Indeed, there has been considerable work on the application of this type of OLs [1,4,5,7,9,13–17], on tilted BCOLs [5,3], and its effects on superfluidity [18], where a rational approximation to the real irrational wavelength ratio has been considered. Yet to what extent a rational ratio would influence the dynamics of a BEC in this type of lattice has, to the best of knowledge, not been explored. (In the latter case, one talks about a BCOL with quasiperiodicity.) As such, the current investigation is for the purpose of examining the effects of a quasiperiodic BCOL on the mobility of the bosons within the framework of a standard mean-field approach. The system considered is a BEC confined in a BCOL superimposed on an external harmonic trap. The long-time evolution of the BEC is particularly explored by computing the averages of a number of physical observables over extremely long times. Long-time dynamics have been examined earlier by, e.g., Min et al. [19] for a BEC expanding in a speckle or impurity disorder potential.

That said, it should be mentioned that the dynamics of a BEC has been explored earlier in a periodic as well as a superlattice [1]. In addition, the localization of an expanding BEC has been examined in a disordered potential [20]. The latter focused on the regime where the interactions dominate over the kinetic energy and disorder; a regime that is considered in this work as well. In this regard, it should be mentioned that there has been considerable work supporting the relevance of disorder on BEC systems [6,8,21–24], on tilted BCOLs [5,3]. The dynamics of BECs in quasidisordered potentials is also attracting significant attention in a quest for observing Anderson localization in BECs without interactions or with repulsive interactions [20]. Other work on this topic includes the investigations of localization [6,25,2], parametric excitations in a combined OL and harmonic trap [26], and

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the formulation of an exact analytical model for the dynamics of a BEC in a BCOL [9]. In Ref. [9], it has been found that the overlap of two OLs of different depths and incommensurate wavelengths results in geometrical frustration of the BCOL. As the depth of the latter rises, the lattice frustration increases which allows more inter-site tunneling of the BEC. Therefore, questions of the following type arise: Is it possible that in a combined harmonic plus BCOL potential an increased lattice frustration leads to a higher kinetic energy? Perhaps a rise in the secondary OL depth allows more tunneling between the lattice sites? How strong is the influence of the harmonic trap as compared to the BCOL in determining the dynamic behavior of the BEC? These are questions that shall be tackled in this paper. Of importance is the interplay between the BCOL and the interactions in the degree of localization. What is significant in this work here, is that the role of the external harmonic trap is added to this interplay.

Next to this, it should be mentioned that the dynamics of BECs in traps excited by moving obstacles have led to a surge of investigations [27–41]. The research on this is even taking further dimensions such as the examination of dynamical fragmentation in a BEC [42]. To this end, mostly repulsive obstacles [32,43] have been obtained experimentally by a blue-detuned laser beam [27,32,35,40]. Here an attractive obstacle is considered which is generated by a red-detuned laser [33,34,44–53] to investigate whether a combined harmonic plus BCOL potential is able to suppress the effects arising from the added energy levels of the red laser potential well.

Further, a comparison is made between the effects of a BCOL with a rational and an irrational ratio of the constituting wavelengths on the long-time dynamics. An investigation most relevant to the present work has been undertaken by Cataliotti et al. [15], who examined the dynamics of a BEC in a combined harmonic and periodic OL potential with quite interesting results. The latter investigation has motivated us to undertake the present work.

The key results of the present work are as follows: (1) it is demonstrated that the dynamics are not influenced by the differences arising from a rational or an irrational ratio of the wavelengths as the harmonic trap overcomes them. In this regard, there is also no effect arising from the lattice frustration; (2) at low primary OL-depths V_0, the long-time averaged physical observables of the BEC do not change with an increase of the secondary-OL depth V_1. However, for a larger V_0 beyond a certain value, V_1 begins to assert itself. In the latter case, the band structure of the system begins to change significantly under the effects of an increased lattice frustration; (3) the effects of a secondary OL in the presence of a primary one of a high intensity ($\sim 100\hbar\omega_{ho}$) induce modulations in the BEC wavefunction whose effects are manifested through the kinetic term.

The organization of the present paper is as follows. In Section 2 the method is briefly outlined. In Section 3 the results are presented and discussed. In Section 4 the paper concludes with some closing remarks.

2. Method

The provenance of the numerical method used in this work is found in Ref. [54]. As for the setup of the current system, it has been explained earlier in previous work [55–57] and the reader is referred to them for details. The system considered is an interacting BEC of ^{87}Rb atoms at $T = 0$ K confined in a harmonic trap that is surrounded by a box potential.

2.1. Gross–Pitaevskii equation

Essentially, the time-dependent 1D Gross–Pitaevskii equation (GPE) in units of the trap [54,56] is given by

$$i\frac{\partial\varphi(x;t)}{\partial t} = \left[-\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} + V_{ho}(x) + V_{LP}(x;t) + V_{OL}(x) + \mathcal{G}|\varphi(x;t)|^2\right]\varphi(x;t), \tag{1}$$

where the coordinate x is rescaled so that $x \to x/\sqrt{2}$ as in Ref. [54]. It is solved numerically via the split-step Crank–Nicolson (CN) method [54] in real time. The accuracy of the CN simulation has been established in Ref. [2]. In Eq. (1), $V_{ho}(x) = \sigma x^2/4$ is the harmonic oscillator potential with σ being its strength,

$$V_{LP}(x;t) = A\exp[-\epsilon(x - vt)^2] \tag{2}$$

the laser potential (LP) with A being its depth, v its velocity, and ϵ the parameter describing its width. $\varphi(x;t)$ is the GP wavefunction, $V_{OL}(x)$ the BCOL described below, and $\mathcal{G}$ the nonlinear interaction parameter given by [54]

$$\mathcal{G} = 2a_s N\sqrt{2\lambda\kappa}/\ell, \tag{3}$$

with λ and κ being the anisotropy parameters describing the width of the harmonic oscillator ground state wavefunction in the y and z directions, respectively, a_s the s-wave scattering length, N the number of particles, and $\ell = \sqrt{\hbar/(m\omega_{ho})}$ is a length scale so that $a_{ho} = \ell/\sqrt{2}$ is the trap length, where m is the mass of a boson and ω_{ho} the trapping frequency. It is understood that (1) was obtained from the 3D GPE after integrating out the y and z dependences [54]. For the present purpose, we set $\lambda = \kappa = 100$ in order to obtain an exactly 1D system. The wave function φ is normalized to 1. The bounding box has a size $2L$ so that $-L \le x \le +L$, and in order to enforce the boundary conditions we set $\varphi(x = \pm L; t)$ and $d\varphi(x;t)/dx|_{x=\pm L} = 0$. The units are explained in Section 2.5.

2.2. Bichromatic optical lattice

The BCOL is generated by

$$V_{OL}(x) = V_0\cos^2(\alpha\pi x) + V_1\cos^2(\beta\pi x), \tag{4}$$

where V_0 is the primary, and V_1 the secondary OL-depth. The parameters α and β determine the periodicity of the OL, that is, whether there is quasiperiodicity or quasidisorder. A measure for the strength of quasidisorder can be obtained by computing the standard deviation $\delta V = \sqrt{\langle V^2\rangle - \langle V\rangle^2}$ where

$$\langle V\rangle = \frac{1}{2L}\int_{-L}^{+L}[V_{OL}(x) + V_{ho}(x)]\,dx, \tag{5}$$

and

$$\langle V^2\rangle = \frac{1}{2L}\int_{-L}^{+L}[V_{OL}(x) + V_{ho}(x)]^2\,dx. \tag{6}$$

Hence, δV determines a degree of disorder largely influenced by the external harmonic trap $V_{ho}(x)$.

2.3. Physical observables

The physical observables that we shall be looking at are the zero-point energy

$$E_{zp}(t) = \int_{-L}^{+L} dx\left[\frac{\partial}{\partial x}|\varphi(x;t)|\right]^2, \tag{7}$$

the kinetic energy of superflow

$$E_{flow}(t) = \int_{-L}^{+L} dx\left[\frac{\partial}{\partial x}\phi(x;t)\right]^2|\varphi(x;t)|^2, \tag{8}$$

$\phi(x;t)$ being the phase of the BEC, the interaction energy