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Hall effect in hopping regime

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ABSTRACT

A simple description of the Hall effect in the hopping regime of conductivity in semiconductors is presented. Expressions for the Hall coefficient and Hall mobility are derived by considering averaged equilibrium electron transport in a single triangle of localization sites in a magnetic field. Dependence of the Hall coefficient is analyzed in a wide range of temperature and magnetic field values. Our theoretical result is applied to our experimental data on temperature dependence of Hall effect and Hall mobility in ZnO.

1. Introduction

The dc Hall effect (HE) in the hopping regime of conductivity was addressed theoretically a number of times in the past [1–7], and various mathematical expressions for the Hall coefficient R_H and Hall mobility μ_H were proposed. However, to our knowledge a well established description of HE in the hopping regime does not exist. Most theories of the hopping HE are based on calculation of three-site-jump probabilities [1] and subsequent construction of an equivalent resistor network in order to calculate the electron transport coefficients. Here we present a different and simpler derivation of the expression for the Hall coefficient and Hall mobility in the case of hopping conductance. We derive an expression for the transverse Hall-current based on the existing theory of two-site jump rate and then, from the balance of the Hall-current and transverse (compensating) electric-field-current, we derive the expressions for R_H and μ_H.

Our paper is organized as follows. We first calculate difference of jump probabilities and Hall current I_{zm} in magnetic field (Section 2). In Section 3 we express I_{zm} using the known formula of two-site transition rate. Then in Section 4 we derive the expression for R_H and μ_H. Comparison with experiment is given in Section 5.

2. Tunneling probabilities

The Hall effect in the hopping regime is not caused by the Lorentz force, because in the hopping conduction an electron can only follow a limited number of paths defined by the electron localization sites (LS). It is commonly considered that HE in the hopping regime is related to the self-interference effect of the electron wave function which propagates along different hopping paths in the magnetic field. Thus, to observe the interference, at least three localization centers should be taken into account. This mechanism was applied for the first time by Holstein [1], for description of hopping HE.

An electron trapped on a LS can jump to any of its neighbors. We will consider only a fraction of jumps which contribute to the *net* current. Because hopping probability decreases exponentially with the distance [8], let us consider only two closest neighbors of any occupied LS, from the half-space in the direction of the net electron flow. In this manner for most occupied LSs we can construct a triangle of sites, which will contribute to HE. In order to simplify the calculation we are going to average over all possible triangles, and consider an equilateral triangle with the side length Δr equal to average distance between localization sites (Fig. 1a).

In the initial state an electron is located on the site 1 (Fig. 1a). We will calculate the probability to find the electron on the sites 2 and 3 in magnetic field. The difference in the occupation probabilities for the sites 2 and 3 will produce over time a nonuniform charge distribution and the Hall voltage.

We will use for the calculation the Feynman's "probability amplitude" approach [9]. We denote by ψ_{12}, ψ_{13} and ψ_{32} the probability amplitudes of hopping between pairs of sites 1-2, 1-3

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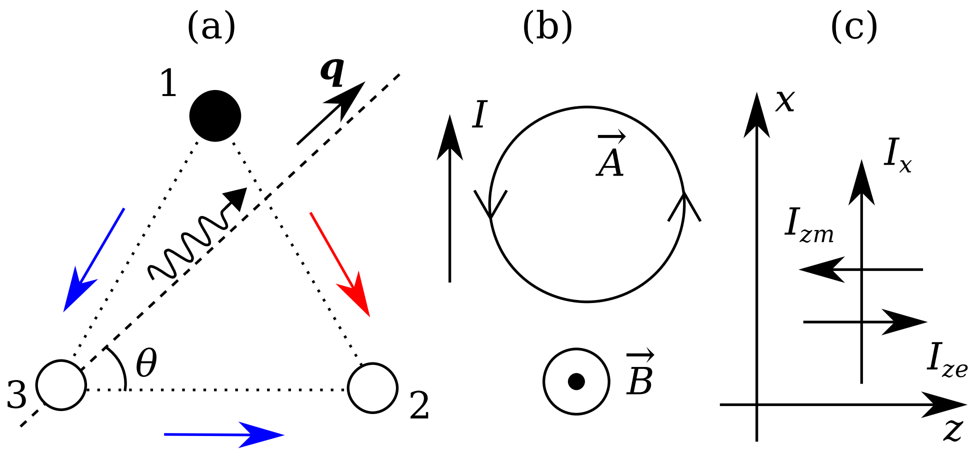

Fig. 1. (a) Averaged triangle of localization sites. The occupied LS is marked by the black circle. Arrows show two possible hopping paths leading to localization site 2. Angle θ describes the direction of propagation of acoustic wave. (b) Direction of the applied electric field (current flow), direction of uniform magnetic field and direction of rotation of the magnetic vector potential. (c) Coordinate system and schematic picture of the considered electric currents.

and 3-2, without any other localization sites around. Let us calculate first the probability of a jump to the right localization site in the considered triangle. The probability amplitude that the electron will be detected on site 2 is

$$\Psi_r = \psi_{12} + \psi_{13}\psi_{32}. \quad (1)$$

The hopping paths, which are taken into account in this formula, are shown in Fig. 1a by arrows. Then the probability of hopping to the right will be

$$P_r = \Psi_r \Psi_r^*. \quad (2)$$

We will represent probability amplitudes in the form $\psi_{ij} = f_{ij} e^{i\varphi_{ij}}$, where f_{ij} and φ_{ij} are real values. The amplitude f_{ij} is a measurable value which is related to the hopping rate and φ_{ij} is the additional phase gained during the transition from one site to another.

Due to the symmetry of the problem there is $f_{12} = f_{13} \equiv f_a$. We denote these values as f_a. There is $f_{32} \equiv f_b < f_a$ because the transition from site 3 to 2 is not facilitated by applied electric field. Thus ψ_{ij} can be written as

$$\psi_{12} = f_a e^{i\varphi_{12}}, \quad \psi_{13} = f_a e^{i\varphi_{13}}, \quad \psi_{32} = f_b e^{i\varphi_{32}}. \quad (3)$$

After substitution of Eq. (3) into Eq. (1) we get

$$\Psi_r = f_a e^{i\varphi_{12}} [1 + f_b e^{i(\varphi_{13} + \varphi_{32} - \varphi_{12})}]. \quad (4)$$

Phases φ_{ij} can be written [9] as $\varphi_{ij} = \varphi_{ij0} + \varphi_{ijH}$, where φ_{ij0} is the phase gained in the absence of magnetic field, and the φ_{ijH} is the phase change induced by magnetic field. The expression for φ_{ijH} is [9]

$$\varphi_{ijH} = -\frac{e}{\hbar} \int_i^j \boldsymbol{A} d\boldsymbol{r}, \quad (5)$$

and the expression in the exponent in Eq. (4) can be written as

$$\varphi_{13} + \varphi_{32} - \varphi_{12} = \beta - \frac{e}{\hbar} \left\{ \int_1^3 \boldsymbol{A} d\boldsymbol{r} + \int_3^2 \boldsymbol{A} d\boldsymbol{r} - \int_1^2 \boldsymbol{A} d\boldsymbol{r} \right\}. \quad (6)$$

Here β stands for $\varphi_{130} + \varphi_{320} - \varphi_{120}$. The sum from the parentheses can be transformed into

$$\int_1^3 \boldsymbol{A} d\boldsymbol{r} + \int_3^2 \boldsymbol{A} d\boldsymbol{r} + \int_2^1 \boldsymbol{A} d\boldsymbol{r} = \oint_{1\to3\to2\to1} \boldsymbol{A} d\boldsymbol{r} = BS = \Phi. \quad (7)$$

In the last transformation we have used Stokes' theorem and relation $\nabla \times \boldsymbol{A} = \boldsymbol{B}$. Since we assume a uniform magnetic field, the integral will result in the product BS, where B is the magnetic induction and S is the surface area outlined by the integration path $1 \to 3 \to 2 \to 1$. It is not obvious how this path looks like, but it is logical to assume that tunneling occurs along the shortest path. In such case S is the area of the triangle. Then, the expression in the parentheses equals to the magnetic flux through the triangle formed by three localization sites. Since the direction, followed by the integration path, coincides with the direction of rotation of the vector potential $\boldsymbol{A}$ (see Fig. 1a, b), the value of the integral is positive and hence $\varphi_{13H} + \varphi_{32H} - \varphi_{12H} = -(e/\hbar)\Phi < 0$.

After the substitution of Eqs. (7)→(6)→(4)→(2), we obtain the probability of electron jumping to the right

$$P_r = f_a^2 \left[1 + f_b^2 + 2f_b \cos\left(\beta - \frac{e}{\hbar}\Phi \right) \right]. \quad (8)$$

Since $-(e/\hbar)\Phi < 0$, the total phase in Eq. (8) will decrease and hence P_r will increase with the increasing magnetic field.

A similar consideration of the probability of electron jumping to the left (localization site 3) gives

$$P_l = f_a^2 \left[1 + f_b^2 + 2f_b \cos\left(\beta + \frac{e}{\hbar}\Phi \right) \right]. \quad (9)$$

This shows that the phase will increase and P_l will decrease with the increasing magnetic field. Thus, electron will tend to deviate more often to the right, which coincides with the direction of deviation that would be induced by the Lorentz force.

In order to obtain normalized probabilities we introduce two new quantities

$$P_{rn} = \frac{P_r}{P_r + P_l}, \quad P_{ln} = \frac{P_l}{P_r + P_l}. \quad (10)$$

These are relative probabilities, indicating the chance to find the electron on the right or left site, if the transition has already occurred. For the normalized values there is $P_{rn} + P_{ln} = 1$ and, in the absence of magnetic field, $P_{rn}(0) = P_{ln}(0) = \frac{1}{2}$, as one would expect in the symmetric system.

The Hall voltage appears due to asymmetry between the probabilities of jumps to the right and left side of the triangle, and it is proportional to

$$P_{rn} - P_{ln} = \frac{P_r - P_l}{P_r + P_l} = \frac{2f_b \sin\beta \sin\left(\frac{e}{\hbar}\Phi\right)}{1 + f_b^2 + 2f_b \cos\beta \cos\left(\frac{e}{\hbar}\Phi\right)}. \quad (11)$$

In the Appendix we show, that $\beta = \pi/2$, and hence this expression is simplified to