Physica B 483 (2016) 37–43

Contents lists available at ScienceDirect

Physica B

journal homepage: www.elsevier.com/locate/physb

Dynamics of a Landau–Zener transitions in a two-level system driven by a dissipative environment

M.E. Ateuafack [a], J.T. Diffo [a,b,*], L.C. Fai [a]

[a] Mesoscopic and Multilayer Structures Laboratory, Department of Physics, Faculty of Science, University of Dschang, Cameroon
[b] Department of Physics, Higher Teachers' Training College, The University of Maroua, PO Box 55 Maroua, Cameroon

ARTICLE INFO

Article history:
Received 16 September 2015
Received in revised form
27 November 2015
Accepted 9 December 2015
Available online 9 December 2015

PACS:
03.65.Yz
75.10.Jm
03.75.Gg
03.67.Ac

Keywords:
Two-level quantum system
LZ transition probability
Dissipative environment
Dephasing
Ohmic bath coupling
Decoherence rate

ABSTRACT

The paper investigates the effects of a two-level quantum system coupled to transversal and longitudinal dissipative environment. The time-dependent phase accumulation, LZ transition probability and entropy in the presence of fast-ohmic, sub-ohmic and super-ohmic quantum noise are derived. Analytical results are obtained in terms of temperature, dissipation strength, LZ parameter and bath cutoff frequency. The bath is observed to modify the standard occupation difference by a decaying random phase factor and also produces dephasing during the transfer of population. The dephasing characteristics or the initial non-zero decoherence rate are observed to increase in time with the bath temperature and depend on the system-bath coupling strength and cutoff frequency. These parameters are found to strongly affect the memory and thus tailor the coherence process of the system.

1. Introduction

A large number of quantum phenomena can be investigated with the help of a two-level system. In the absence of environmental coupling, two-level quantum systems can experience an avoided crossing in time [1–4] and is generally known as the Landau–Zener (LZ) problem. The quantities of interest are commonly the population probabilities for both the ground and the excited diabatic states at exceedingly long time intervals far from the crossing region. Such a universal phenomenon is ubiquitous and have applications in quantum electrodynamics [5,6], spin-flip in nanomagnets [7], Bose–Einstein condensates in optical lattices [8] and adiabatic computing [9–19] where the environment is considered as a source of classical noise [20–23], or quantum fluctuations that lead to transitions between qubit states [16,**?**,25–34] or pure dephasing [35,36]. Environment-driven LZ transitions have been applied for the optimal implementation of an adiabatic quantum computing algorithm given the promise of adiabatic quantum computing as an alternative approach to achieve extremely high-speed computation [37,38].

Environmental parameters, such as temperature, are found to exponentially enhance the coherent oscillations generated within a LZ transition [30]. Specifically, it has been shown that the occupation probability of a two-level system coupled to harmonic oscillation exhibits a nonmonotonic dependence on the coupling strength and temperature [39].

In Ref. [40] the authors interpret the LZ transition as an interference process and demonstrate the enhancement of the asymptotic transition probability due to the phase-uncertainty factor. On the other hand, the transition probability transition of the two-levels in the LZ model is suppressed by relaxation events due to energy dissipation of the system. Thus dephasing and relaxation may have competing effects on the dynamics [41,42]. Hence, coherence times are very limited in two-level quantum systems preventing the observation of Rabi oscillations. However, these oscillations have been experimentally observed as a result of multiple LZ processes (more than 100) in a single electronic spin system [43].

In this paper we investigate the effects of a two-level quantum system coupled to an ohmic, sub-ohmic and super-ohmic environment. The system–bath interaction is of spin-boson model

* Corresponding author at: Department of Physics, Higher Teachers' Training College, The University of Maroua, PO Box 55 Maroua, Cameroon.
E-mail addresses: esouamath@yahoo.fr (M.E. Ateuafack), diffojaures@yahoo.com (J.T. Diffo), corneliusfai@yahoo.fr (L.C. Fai).

http://dx.doi.org/10.1016/j.physb.2015.12.012
0921-4526/

type with longitudinal and transversal couplings. Since the asymptotic value of the LZ transition probability does not show the temporal evolution of the quantum system, it is necessary to perform detailed analysis on the dynamics of a two-level quantum system in the presence of dissipation. We derive the time-dependent phase accumulation and the LZ transition probability in the presence of fast quantum noise. We measure the coherence dynamics of a two-level quantum system with fast quantum noise.

The paper is organized as follows: in Section 2, we present the dissipative LZ model and the corresponding Hamiltonian. In Section 3, we develop the density matrix approach in a dissipative LZ model and derive the asymptotic LZ transition probability. In Section 4, we derive the time dependent LZ transition probability. Section 5 studies the time dependence coherence of a two-level quantum system in the quantum bath. Section 6 is the conclusion.

2. Model system Hamiltonian

Consider a two-state system originally in the state $|1\rangle$ at time $t = -\infty$ and evolves non-adiabatically in time to a final state $|2\rangle$. If the energy difference between the diabatic states is considered to vary linearly with time then for an isolated system, the LZ Hamiltonian is given by

$$\hat{H}_{LZ}(t) = \beta t\hat{\sigma}_z + J\hat{\sigma}_x. \tag{1}$$

Here, $v = 2\beta > 0$ is the velocity; σ_ℓ, $\ell = x, z$, Pauli matrices and J the LZ gap.

Suppose our system is subjected to a quantum bath with the total Hamiltonian

$$\hat{H}(t) = \hat{H}_{LZ}(t) + \hat{H}_{SB} + \hat{H}_B, \tag{2}$$

where

$$\hat{H}_{SB} = \sum_{\ell,k} \hat{\sigma}_\ell \gamma_k\left(\hat{a}_k + \hat{a}_k^\dagger\right), \quad \ell = (x, y, z) \tag{3}$$

is the interaction Hamiltonian,

$$\hat{H}_B = \sum_k \omega_k \hat{a}_k^\dagger \hat{a}_k \tag{4}$$

the bath Hamiltonian; γ_k, the coupling strength and where $\hat{a}_k$ and $\hat{a}_k^\dagger$ are respectively the phonon annihilation and creation operators. We assume the bath to be in thermal equilibrium at temperature T and exceedingly larger than the system thus leading to Gaussian fluctuations. Therefore, the average over the ensemble of classical functions can be replaced by the thermal expectation value. Consider the following unitary transformation with respect to the bath:

$$U(t) = \exp\{-i\int H_B\, dt'\}. \tag{5}$$

Then in the interaction picture, the total Hamiltonian in Eq. (2) reads:

$$\hat{H}(t) = (\beta t + f_k(t))\hat{\sigma}_z + (J + f_k(t))\hat{\sigma}_x + f_k(t)\hat{\sigma}_y. \tag{6}$$

Here the function $f_k(t)$ characterizes the environment and obeys the Markovian Gaussian process specified by the following two-time correlation function:

$$\langle f_k(t_1) f_k(t_2)\rangle = \sum_k |\gamma_k|^2\left[n_k \exp\{i\omega_k(t_1 - t_2)\} + (n_k + 1)\exp\{-i\omega_k(t_1 - t_2)\}\right], \tag{7}$$

with

$$\langle f_k(t)\rangle = 0. \tag{8}$$

Here, $\langle\cdots\rangle$ denotes the thermal expectation value and

$$n_k = \frac{1}{2}\left[\coth\left(\frac{\omega_k}{2T}\right) - 1\right] \tag{9}$$

are the average phonon occupation numbers. The relevant aspect of Eq. (6) corresponds for instance to spin frustration by hyperfine field or Overhauser field.

3. Density matrix approach in a dissipative environment

Theoretically, the important object for the investigation of the system's dynamics is the probability $P(t)$ of finding the system at time t. In the Linblad axiomatic formalism the irreversible time evolution of a system is described by the following general quantum Markovian master equation for the density operator [44]

$$i\frac{d\hat{\rho}(t)}{dt} = \left[\hat{H}(t), \hat{\rho}(t)\right], \quad \hat{\rho}(t) = \begin{bmatrix}\rho_{11} & \rho_{12}\\ \rho_{21} & \rho_{22}\end{bmatrix}. \tag{10}$$

From here we arrive at the following system of differential equations for the two-level system:

$$\begin{aligned} i\dot{\rho} &= 2f_k^-(t)\rho_{21} - 2f_k^+(t)\rho_{12}\\ i\dot{\rho}_{12} &= 2(\beta t + f_k(t))\rho_{12} - f_k^-(t)\rho\\ i\dot{\rho}_{21} &= -2(\beta t + f_k(t))\rho_{21} - f_k^+(t)\rho, \end{aligned} \tag{11}$$

where $\rho = \rho_{11} - \rho_{22}$, $f_k^\pm(t) = J + (1 \pm i)f_k(t)$ with

$$\langle f_k^\pm(t_1) f_k^\pm(t_2)\rangle = J^2 + (1 \pm i)^2 \sum_k |\gamma_k|^2\left[n_k \exp\{i\omega_k(t_1 - t_2)\} + (n_k + 1)\exp\{-i\omega_k(t_1 - t_2)\}\right] \tag{12}$$

and

$$\langle f_k^-(t_1) f_k^+(t_2)\rangle = \langle f_k^+(t_1) f_k^-(t_2)\rangle = J^2 + 2\sum_k |\gamma_k|^2\left[n_k \exp\{i\omega_k(t_1 - t_2)\} + (n_k + 1)\exp\{-i\omega_k(t_1 - t_2)\}\right]. \tag{13}$$

The integral–differential equation for the occupation difference ρ is obtained from Eq. (12):

$$\frac{d\rho(t)}{dt} = -2\int_{-\infty}^{t} dt_1\left[\exp\left\{-2i\int_{t_1}^{t}\left(\beta t_2 + f_k(t_2)\right)dt_2\right\} f_k^+(t) f_k^-(t_1) + \exp\left\{2i\int_{t_1}^{t}\left(\beta t_2 + f_k(t_2)\right)dt_2\right\} f_k^-(t) f_k^+(t_1)\right]\rho(t_1). \tag{14}$$

Following Ref. [22], the solution of Eq. (14) can be written in the form of a series, considering the initial condition $\rho(t_1) = 1$:

$$\rho(t) = 1 + \sum_{r=1}^{\infty}(-2)^r \int_{-\infty}^{t} dt_1 \int_{-\infty}^{t_1} dt_2 \times \cdots \times \int_{-\infty}^{t_{2r-2}} dt_{2r-1}\int_{-\infty}^{t_{2r-1}} dt_{2r} \prod_{q=r}^{1}\left[A(t_{2q}, t_{2q-1}) + A^*(t_{2q}, t_{2q-1})\right] \tag{15}$$

where the indices are non-zero positive integer and, $A(t, t_1) = \exp\{-i\Omega(t, t_1)\} f_k^+(t) f_k^-(t_1)$, $\Omega(t, t_1) = 2\int_{t_1}^{t}(\beta t_2 + f_k(t_2)\, dt_2$. Considering Gaussian statistics we express the correlator as in Ref. [20]:

$$\left\langle \exp\left\{-2i\int_{t_1}^{t} f_k(t_2')\, dt_2'\right\} f_k^+(t_2) f_k^-(t_1)\right\rangle = \exp\left\{-\int_{t_1}^{t} dt_2 \int_{t_1}^{t} dt_2'' \langle f_k(t_2') f_k(t_2'')\rangle\right\}\langle f_k^+(t_2) f_k^-(t_1)\rangle. \tag{16}$$

From here, we evaluate the thermal expectation value of Eq. (16) applying Wick's theorem considering time ordering: