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Decoherence of Josephson charge qubit in non-Markovian environment

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ABSTRACT

In this paper we investigate the decoherence of Josephson charge qubit (JCQ) by using a time-nonlocal (TNL) dynamical method. Three kinds of environmental models, described with Ohmic, super-Ohmic, and sub-Ohmic spectral density functions are considered. It is shown that the TNL method can effectively include the non-Markovian effects in the dynamical solutions. In particular, it is shown that the sub-Ohmic environment has longer correlation time than the Ohmic and super-Ohmic ones. And the Markovian and non-Markovian dynamics are obviously different for the qubit in sub-Ohmic environment.

1. Introduction

Among various realizations of solid state qubit that is based on the quantum circuits are considered to be promising candidates because of their scalability, established fabrication techniques, and flexibility in designs. In the past decades, many kinds of these qubit models are introduced. They are Josephson charge, [1] flux, [2] phase [3] qubits, and so on. Decoherence [4,5] of these qubits have been widely investigated because it is considered to be a major impediment for taking the qubit as the cell for storing and processing information in future quantum computers. Many efforts in theories [6–12] as well as in experiments [13–15] have been contributed to search for decoherence time as well as decoherence mechanism for the qubit systems. Makhlin et al. [6] using the Fermi golden rule and Bloch type master equation investigated the relaxation and decoherence times for the qubits. And many other works have also been contributed to this problem recently [16–19].

In many investigations of the decoherence for these qubits, the Markovian approximation is always appealed to. It has been found that in some cases with low environmental temperature, this approximation may distort quantum dissipative dynamics, and obtain incorrect decoherence landscape. So it is interested to investigate the decoherence of the Josephson-junction qubits beyond Markovian approximation. In this paper we investigate the coherence dynamics of the Josephson charge qubit (JCQ) by solving the quantum dissipative dynamics by using a non-Markovian approach, the time-nonlocal (TNL) quantum dissipative dynamical method.

A direction for solving the quantum dissipative dynamics including non-Markovian effects is based on the path integral formalism [20,21]. Marki and co-workers have developed an exact numerical dynamics approach—quasi-adiabatic propagator path integral (QUAPI) method [22,23]. This is an efficient way of including memory effects via tensor products of element memory kernels. A different approach was proposed by Tanimura and coworkers, who introduced a hierarchical treatment of non-Markovian dynamics, in which the primary density operator is coupled to auxiliary density operators, describing the effects of successively higher order system–bath interaction [24,25]. These two kinds of non-Markovian dynamical methods can include environmental memory effects, and are extensively used in the investigations of decoherence, disentanglement, energy transformation, and spectral analysis in quantum systems. Many other methods have also been proposed and used in last years [26,27]. However, to discover other non-Markovian dynamical schemes is still expected, because in the existing methods of solving reduced dynamics, huge analytical and/or numerical effort should be made.

For the systems of Hamiltonian with continuous variables, the Langevin equation is an excellent choice [28,29] to solve the reduced dynamics. For the system with several discrete levels, Meier and Tannor recently proposed a new dynamics method, which is based on the time-nonlocal Nakajima–Zwanzig projection operator formalism [30]. This method is similar to the hierarchical approach. In the both approaches the non-Markovian effects are described by the coupling equations of the primary and auxiliary

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density matrices, and the spectral density functions of baths must be parametrized into special forms. Therefore, the parametrization of the spectral density of bath is the key step in use of these approaches. The hierarchical approach has been developed and applied in many fields recently [31,32]. Meier and Tannor have parametrized the Ohmic spectral density, and the dynamics of a two-level open system has been investigated by using the TNL method [30].

It is known that according to the difference of damping strength, one should use the Ohmic, or super-Ohmic, or sub-Ohmic spectral density functions to describe the baths of the systems coupled. The parametrization of the super-Ohmic spectral density function has also been obtained in Ref. [33]. To the best of our knowledge, there have been no previous reports of the parametrization of the sub-Ohmic spectral density function. In order to expand the range of use of the TNL methods, in this paper, we shall investigate the parametrization of not only the Ohmic, super-Ohmic, but also the sub-Ohmic spectral density functions, and use the results to investigate the coherence evolutions of the JCQ. It will be shown that a four steps pretreatment can accelerate the simulated annealing algorithm for the parametrization of the baths. Thus, we can obtain three sets of parameters to model sub-Ohmic, Ohmic, and super-Ohmic spectral density functions. And the parametrization forms are exactly consistent with their original spectral density functions. It will be seen that the Markovian approximation is good enough for the Ohmic and super-Ohmic bath environments. However, this approximation distort the dynamics of the JCQ in sub-Ohmic bath environment. It means that when the environment must be described with sub-Ohmic bath the non-Markovian effects should not be neglected, and this effects can be included in the TNL method.

2. Parametrization of spectral density functions

One of the key steps of the TNL dynamical method is to parametrize the environment spectral density function into the following form:

$$J'(\omega)=\frac{\pi}{2}\sum_{k=1}^{n}p_k\frac{\omega}{[(\omega+\Omega_k)^2+\Gamma_k^2][(\omega-\Omega_k)^2+\Gamma_k^2]}, \tag{1}$$

with arbitrary real parameter sets p_k, Ω_k, Γ_k. In Ref. [30], Meter and Tannor parametrized the Ohmic spectral density function, and obtained three sets of parameters p_k, Ω_k, Γ_k, which make the Ohmic spectral density function $J(\omega)=\eta\omega e^{-\omega/\omega_c}$ exactly be parametrized into Eq. (1). As pointed out in Ref. [30] that, for the given sets of parameters p_k, Ω_k, Γ_k, the parametrization form of the spectral density function is unique. However, one might find out different sets of parameters p_k, Ω_k, Γ_k from one spectral density function due to use of different simulation algorithms. Nevertheless, as long as the parametrization results are consist with the original one, they will not lead a different dynamical behavior, since the dynamics is entirely determined by $J(\omega)$ other than its details. The only important matter for the choice of the simulation algorithm is that the $J'(\omega)$ and $J(\omega)$ must be in agreement with each other in enough large range of frequency ω. It is clear that to increase the range of ω, the amount of calculations will be increased.

In the following, we propose a pretreatment scheme of parameters before usual using simulated annealing algorithm. In this scheme we take the spectral density function, for example, $J(\omega)$ as the aim function, then find out three sets of parameters p_k, Ω_k, Γ_k. Through the three sets of parameters, we construct $J'(\omega)$ as Eq. (1), and use $J'(\omega)$ to replace $J(\omega)$ in the TNL calculations. This pretreatment scheme include four steps as follows: (1) plotting $J(\omega)$, and finding out the peaks of $J(\omega)$ at some ω (for example ω_1) and the range of ω; (2) choosing $p_1=1$ (for example), and trying Ω_1 and Γ_1 with different values, until the $J'(\omega)$ has peaks at ω_1 as $J(\omega)$ has, in the range of ω obtained in step (1); (3) changing the value of p_1, until $J'(\omega_1)\to J(\omega_1)$; (4) plotting $J'(\omega)$ and finding out the ω (for example ω_2) at which $J'(\omega)$ and $J(\omega)$ have the maximum difference. Then repeat steps (2)–(4) and find out p_2, Ω_2, Γ_2, and p_3, Ω_3, Γ_3, … until $J'(\omega_1)\cong J(\omega_1)$. If needed, we can use the usual simulated annealing algorithm based on above obtained p_k, Ω_k, and Γ_k to find their exact values. This scheme is effective and can exactly parametrize any complex spectral density functions in a small amount of calculations.

Table 1
Parameters used for approximating the given sub-Ohmic, Ohmic, and super-Ohmic spectral density functions.

	Sub-Ohmic	Ohmic	Super-Ohmic
$p_1/(\xi\omega_c^4)$	0.09	2.735	41.40
$p_2/(\xi\omega_c^4)$	0.60	4.83	130.00
$p_3/(\xi\omega_c^4)$	0.70	−20.15	−0.01
$p_4/(\xi\omega_c^4)$	0.30	0.01	−75.0
Γ_1/ω_c	0.50	1.50	2.70
Γ_2/ω_c	1.50	2.00	15.0
Γ_3/ω_c	1.00	6.50	0.30
Γ_4/ω_c	0.80	0.30	6.50
Ω_1/ω_c	0.01	0.10	1.50
Ω_2/ω_c	2.50	1.50	12.00
Ω_3/ω_c	1.50	5.50	0.30
Ω_4/ω_c	0.50	0.30	6.50

By using the above scheme we have parametrized the Ohmic, sub-Ohmic and super-Ohmic spectral density functions, $J_o(\omega)=\frac{\pi}{2}\xi\omega e^{-\omega/\omega_c}$; $J_{sub}(\omega)=\frac{\pi}{2}\xi\omega^{\frac{1}{2}}\omega_c^{\frac{1}{2}}e^{-\omega/\omega_c}$; $J_{super}(\omega)=\frac{\pi}{2}\xi\omega^2\omega_c^{-1}e^{-\omega/\omega_c}$. The respective parameters p_k, Ω_k, Γ_k of the three spectral density functions are listed in Table 1.

$J_o(\omega)$, $J_{sub}(\omega)$, and $J_{super}(\omega)$ and their corresponding $J_o'(\omega)$, $J_{sub}'(\omega)$, and $J_{super}'(\omega)$ are plotted in Fig. 1. It is shown that they are in agreement with each other, respectively. According to the above four steps, less amount of calculations is needed to parametrize the spectral density functions. From Table 1 we see that the values of p_k, Γ_k and Ω_k for the Ohmic and super-Ohmic spectral density functions are different from the values obtained in Refs. [30,33]. However, the parametrization results are in good agreement with their original spectral density functions not only for the Ohmic, and super-Ohmic baths but also for the sub-Ohmic bath.

3. Decoherence of the JCQ

In this section we firstly review the JCQ model and then use the TNL method and parametrization spectral density functions to investigate the coherence dynamics of the JCQ. The single JCQ Hamiltonian is [6]

$$H_R=E_{ch}(n-n_g)^2-E_J\cos\phi. \tag{2}$$

Here, $E_{ch}=e^2/(C_g+C_J)$ is the charging energy with the capacitance of the Josephson tunnel junction C_J, and the gate capacitance C_g; $E_J=I_c\hbar/2e$ is the Josephson coupling energy [9], where I_c is the critical current of the Josephson junction, $\hbar$ the Planck's constant divided by 2π; and e the charge of electron; $n_g=C_gV_g/2e$ is the dimensionless gate charge, where V_g the controllable gate voltage. The number operator n of (excess) Cooper pair on the island, and the phase ϕ of the superconducting order parameter, are quantum