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Presentation and investigation of a new two dimensional heterostructure phononic crystal to obtain extended band gap

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ABSTRACT

In this paper, a new heterostructure phononic crystal is introduced. The new heterostructure is composed of square and rhombus phononic crystals. Using finite difference method, a displacement-based algorithm is presented to study elastic wave propagation in the phononic crystal. In contrast with conventional finite difference time domain method, at first by using constitutive equations and strain-displacement relations, elastic wave equations are derived based on displacement. Then, these forms are discretized using finite difference method. By this technique, components of stress tensor can be removed from the updating equations. Since the proposed method needs less elementary arithmetical operations, its computational cost is less than that of the conventional FDTD method. Using the presented displacement-based finite difference time domain algorithm, square phononic crystal, rhombus phononic crystal and the new heterostructure phononic crystal were analyzed. Comparison of transmission spectra of the new heterostructure phononic crystal with those creating lattices, showed that band gap can be extended by using the new structure. Also it was observed that by changing the angular constant of rhombus lattice, a new extended band gap can be achieved.

1. Introduction

Much attention has been attracted to the phononic crystals in recent years. These non homogenous structures are periodic arrangement of inclusions in an elastically different host material. The frequency ranges that propagation of elastic waves is prohibited are called band gap. By this feature, these crystals can control propagation of elastic waves [1,2]. Since the capability of the crystals arise from phononic band gaps, it is essential to achieve the desired phononic band gaps. Acoustic filter and waveguide are some applications of phononic crystals.

Phononic crystals are studied theoretically and experimentally in several researches [3–6]. To achieve the desired band gap, various works have been done on these crystals such as introduction of nesting phononic crystals [7], construction of a hybrid triangular lattice phononic crystal [8], presentation of twinned-square periodic structures [9] and introduction of hierarchical phononic crystals [10].

Heterostructure phononic crystals have been also investigated in some studies recently such as: presenting of a sandwich type of heterostructure in the 2D square lattice by replacing two columns of circular scatter cylinders with two columns of square scatter cylinders [11], investigation of the interface-guided mode of Lamb waves in a phononic crystal heterostructure plate [12], analyzing of heterostructure phononic crystal waveguides [13], construction of an one-dimensional multilayer phononic heterostructure by combining finite periodic phononic crystals and Fibonacci quasi-periodic phononic crystals [14] and design of a phononic heterostructure by merging two one-dimensional phononic crystals [15].

By combining two-dimensional square and rhombus lattices, a new heterostructure phononic crystal is presented in this paper. Then, square and rhombus phononic crystals and the heterostructure phononic crystal were analyzed using displacement-based finite difference time domain method. Band gap of the new heterostructure phononic crystal was computed and compared with band gaps of its square and rhombus lattices. Effect of angular constant of the rhombus lattice on band gap of the heterostructure phononic crystal was studied too.

2. Model and calculation method

2.1. The model

The proposed heterostructure phononic crystal has been shown in Fig. 1. The proposed heterostructure is composed of square and rhombus phononic crystals. This heterostructure is finite in horizontal direction and extends infinitely in vertical

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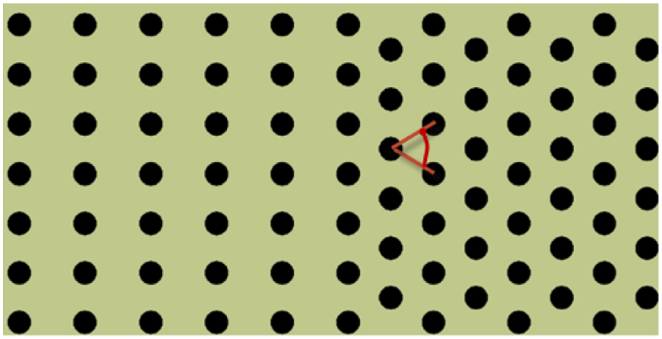

Fig. 1. The proposed hetero-structure phononic crystal that is composed of square arrangement (left) and rhombus arrangement (right).

direction. The relation between constants of square and rhombus lattices can be written as:

$$a' = a/(2\sin(\gamma/2)) \tag{1}$$

where a' and γ are constants of the rhombus lattice and a is constant of square lattice.

2.2. Calculation method

Two-dimensional (2D) phononic crystals are considered here. The formulation can be easily derived for 3D phononic crystals. The inclusions are arranged along the z direction and are repeated in the xy plane. The equations of elastic waves can be written as:

$$\rho\dot{v}_i = \sigma_{ij,j} \tag{2}$$

$$v_i = \dot{u}_i \tag{3}$$

$$\sigma_{ij} = C_{ijmn}u_{m,n} \tag{4}$$

In above equations, $\rho = \rho(x, y)$, $C_{ijkl}(x, y)$, v_i and u_i stand for the density, elastic stiffness tensor, i component of velocity and displacement of the structure, respectively. The summation agreement over dummy indices is considered, too. Since propagation of elastic waves in the xy plane is assumed, the displacement, velocity and stress tensor of the structure are independent of z, i.e., $u_i = u_i(x, y, t), v_i = v_i(x, y, t)$ and $\sigma_{ij} = \sigma_{ij}(x, y, t)$.

Consider that the inclusion and the host materials are isotropic. Then the discretized form of Eqs. (2–4) for study of the mixed mode is given as below:

$$v_1^{l,m;n+1/2} = v_1^{l,m;n-1/2} + \Delta t/\rho^{l,m}\Big[\Big(\sigma_{11}^{l+1/2,m;n} - \sigma_{11}^{l-1/2,m;n}\Big)/\Delta x + \Big(\sigma_{12}^{l,m+1/2;n} - \sigma_{12}^{l,m-1/2;n}\Big)/\Delta y\Big] \tag{5}$$

$$v_2^{l+1/2,m+1/2;n+1/2} = v_2^{l+1/2,m+1/2;n-1/2} + \Delta t/\rho^{l+1/2,m+1/2} \times\Big[\Big(\sigma_{22}^{l+1/2,m+1;n} - \sigma_{22}^{l+1/2,m;n}\Big)/\Delta y + \Big(\sigma_{12}^{l+1,m+1/2;n} - \sigma_{12}^{l,m+1/2;n}\Big)/\Delta x\Big] \tag{6}$$

$$u_1^{l,m;n+1} = u_1^{l,m;n} + v_1^{l,m;n+1/2}\Delta t \tag{7}$$

$$u_2^{l+1/2,m+1/2;n+1} = u_2^{l+1/2,m+1/2;n} + v_2^{l+1/2,m+1/2;n+1/2}\Delta t \tag{8}$$

$$\sigma_{11}^{l+1/2,m;n} = C_{11}^{l+1/2,m}(u_1^{l+1,m;n} - u_1^{l,m;n})/\Delta x + C_{12}^{l+1/2,m}(u_2^{l+1/2,m+1/2;n} - u_2^{l+1/2,m-1/2;n})/\Delta y \tag{9}$$

$$\sigma_{12}^{l,m+1/2;n} = C_{44}^{l,m+1/2}\Big[\Big(u_1^{l,m+1;n} - u_1^{l,m;n}\Big)/\Delta y + \Big(u_2^{l+1/2,m+1/2;n} - u_2^{l-1/2,m+1/2;n}\Big)/\Delta x\Big] \tag{10}$$

$$\sigma_{22}^{l+1/2,m;n} = C_{11}^{l+1/2,m}\Big(u_2^{l+1/2,m+1/2;n} - u_2^{l+1/2,m-1/2;n}\Big)/\Delta y + C_{12}^{l+1/2,m}\Big(u_1^{l+1,m;n} - u_1^{l,m;n}\Big)/\Delta x \tag{11}$$

where (l,m) and n stands for location of a typical node and n-th time step, respectively.

As Eq. (5) reveals, updating of x component of velocity at node (l,m) requires calculation of components of stress tensor at four coordinates, i.e., $\sigma_{11}^{l+1/2,m;n}$, $\sigma_{11}^{l-1/2,m;n}$, $\sigma_{12}^{l,m+1/2;n}$, $\sigma_{12}^{l,m-1/2;n}$. As Eqs. (9–11) implies, computation of each of these stress components needs sizeable elementary arithmetical operations. To calculate transmission spectra of a phononic crystal, Fourier transformation of displacement data are taken.

To compute transmission spectra of a phononic crystal, the calculated stress components have no direct contributions and are only used for updating components of displacement. By obtaining displacement-based forms of elastic wave equations and discretization of resultant equations, components of stress tensor can be eliminated from the updating equations.

This course of action gives efficient updating equations that need less elementary arithmetical operations than conventional FDTD method. So the needed computational cost of the efficient updating equations is less than that of conventional FDTD method. The procedure of obtaining these updating equations is described below.

For points far apart the interface of the inclusion and the host material (elastic constants of the structure are not discontinuous in these points), by substituting divergence of Eq. (4) in Eq. (2) the displacement-based form of elastic wave equation can be derived. By discretization of the resultant equations, displacement-based finite difference time domain (DBFDTD) formulation can be obtained. Consider that the inclusion and the host materials are isotropic. The displacement-based form of elastic wave equations for study of the mixed mode in this case can be given as below:

$$\rho\dot{v}_1 = C_{11}u_{1,11} + C_{44}u_{1,22} + (C_{12} + C_{44})u_{2,12} \tag{12}$$

$$\rho\dot{v}_2 = C_{11}u_{2,22} + (C_{12} + C_{44})u_{1,12} + C_{44}u_{2,11} \tag{13}$$

In Eqs. (12) and (13) C_{11}, C_{12} and C_{44} are elastic constants of an isotropic material. To discretize Eqs. (12) and (13), all derivatives are replaced with central difference estimates. The consequent equations can be applied to update the velocity and displacement of the non interfacial grid points. For the interfacial grid points, the discretized form of the Eqs. (2–4) are used. For a suitable presentation of the discretized form of Eqs. (12) and (13), the subsequent coefficients are defined: