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Rotating ideal Fermi gases under a harmonic potential

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ABSTRACT

We present a numerical analysis on thermodynamics of a harmonically trapped ideal Fermi gases subjected to either rotating frame or synthetic magnetic field. We discuss the rotation frequency dependency of chemical potential, specific heat, magnetization, particle flow and density profile. Our results demonstrate that the magnetization displays three characteristic regions: mesoscopic fluctuation, de Haas–van Alphen oscillation and Landau diamagnetism. The center and amplitude of oscillation peaks in particle flow in rotating frame exhibit much stronger dependence on rotation frequency than those in synthetic magnetic field.

1. Introduction

Recently, an ideal, inhomogeneous, quantum many-body Fermi system is realized in cold atomic system [1–3]. Although perhaps not as dramatic as the phase transition associated with bosons, the thermodynamic behavior of trapped Fermi gases also merits a renewed attention. Among these studies, a trapped Fermi gas subject to a rotation is an interesting issue due to its stability [4]. Earlier proposal is to rotate atomic gases in a rotating trap, which is called rotating frame method [5]. This is similar to the rotating-bucket experiment of superfluid helium. More recently, synthetic magnetic field approach is developed in experiments [6,7]. This approach creates an effective gauge potential for neutral cold atoms by means of an optical field and thus makes neutral atoms behave like charged particles in a magnetic field. By choosing the proper "magnetic field", one can simulate astrophysical scenarios and observe physical effects, such as formation of non-Abelian magnetic monopoles for cold atoms [8].

An advantage of ultracold Fermi system is its high purity and controllability, which is needed in conventional Fermi systems, such as electrons in atoms, electron gases in the solid-state context and metals, and nucleons in nuclei [9–14]. Recently, many experimental and theoretical groups have studied the magnetic response of electrons in metals under various confining potentials, which lead to the generation of de Haas–van Alphen-type oscillations in persistent current and consequently orbital magnetism [11–13]. However, the directly experimental observation on these quantum oscillations in current and orbital magnetism for constrained electron system becomes rather difficult due to the bulk characteristics. On the contrary, rotating Fermi gas can offer researchers an opportunity to investigate many-body physics of confined electrons.

In this paper, we propose a theoretical scheme to mimic the motion of confined electrons under a magnetic field in a fast rotating background and observe these quantum oscillations in physical observable by use of rotating ideal Fermi system [4,15]. Based on their similar Hamiltonians between confined electrons and rotating neutral fermions, we believe that such interesting magnetic behavior of confined electrons is expected to exhibit in rotating Fermi gases. We study thermodynamics of harmonically trapped ideal Fermi gases in two rotary modes (synthetic magnetic field and rotating frame). One main concern of our work is to show how the oscillatory behaviors in fermions depend on the temperature and rotary modes.

The paper is organized as follows. In Section 2, we give a short review of model and derive the thermodynamic expressions. We discuss the low and high temperature limits of chemical potential and specific heat based on semi-classical approximation. In Section 3, we present the corresponding numerical results and make an elaborate comparison with two rotary modes. We emphasize the similarities but also the key differences between synthetic magnetic field and rotating frame. In Section 4, we calculate the particle flow distribution in rotating Bose gases to compare with that for Fermi case. In the last section, we give a brief summary of our main results.

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2. Model and expressions of thermodynamic quantities

2.1. Trapped Fermi gases in synthetic magnetic field

We consider a spinless fermion of mass M moving in a harmonic trap under a synthetic magnetic field $\mathbf{B} = \mathbf{e}_z B$ with effective Hamiltonian

$$\hat{H} = \frac{\mathbf{P}^2}{2M} + \frac{1}{2}M\left(\omega_0^2 + \omega_L^2\right)\left(x^2 + y^2\right) + \frac{1}{2}M\omega_z^2 z^2 - \omega_L \hat{L}_z = \frac{\left(\mathbf{P} - \mathbf{A}\right)^2}{2M} + \frac{1}{2}M\omega_0^2\left(x^2 + y^2\right) + \frac{1}{2}M\omega_z^2 z^2. \quad (1)$$

$\mathbf{P}$ is the momentum operator, and $\hat{L}_z$ is the orbital angular momentum operator. ω_0 and ω_z represent the transverse and axial frequencies of harmonic potential, respectively. ω_L denotes Larmor frequency.

Note that Eq. (1) looks like that of trapped charged particles in a magnetic field with gauge potential $\mathbf{A} = (\mathbf{B} \times \mathbf{r})/2$ [16]. Therefore, per-particle eigenvalue can be written in the dimensionless form

$$\bar{\varepsilon} = \varepsilon_{nkm}/\hbar\omega_z = n + \frac{1}{2} + \left(2k + |m| + 1\right)\sqrt{\alpha^2 + \bar{B}^2} - m\bar{B}, \quad (2)$$

where $n = 0, 1, 2, \ldots$, $k = 0, 1, 2, \ldots$ are non-negative integers, and $m = 0, \pm 1, \pm 2, \ldots$ is the angular quantum number. Aspect ratio of harmonic trap and dimensionless synthetic magnetic field are defined by $\alpha = \omega_0/\omega_z$ and $\bar{B} = \omega_L/\omega_z$, respectively.

The corresponding per-particle eigenfunction in the cylindrical coordinate is

$$\psi_{nkm}(\rho, \varphi, z) = \frac{e^{im\varphi}}{\sqrt{2\pi}} R_{km}(\rho) Z_n(z), \quad (3)$$

where the radial wave function

$$R_{km}(\rho) = \frac{\sqrt{2}}{l}\left[\frac{k!}{(k + |m|)!}\right]^{1/2}\left(\frac{\rho}{l}\right)^{|m|}\exp\left(-\frac{\rho^2}{2l^2}\right)L_k^{|m|}\left(\frac{\rho^2}{l^2}\right), \quad (4)$$

and the nth harmonic oscillator eigenfunction

$$Z_n(z) = \pi^{-1/4}\left[\frac{1}{2^n n!}\right]^{1/2}\xi^{-1/2}\exp\left[-\frac{1}{2}\left(\frac{z}{\xi}\right)^2\right]H_n\left(\frac{z}{\xi}\right). \quad (5)$$

$L_k^{|m|}$ is the Laguerre polynomial. The characteristic lengths are defined by $l = \sqrt{\hbar/M\omega}$ and $\xi = \sqrt{\hbar/M\omega_z}$ with $\omega = \sqrt{\omega_0^2 + \omega_L^2}$.

This system is comprised of a Fock–Darwin-like state in xy plane and an eigenstate of harmonic oscillator in the z direction. The rotating symmetry of the harmonic trap is more incorporated in the cylindrical coordinate [12–14,17,18]. Eqs. (2) and (3) are valid for harmonically trapped particles, including electrons with a finite magnetic field [12–14], and neutral fermions or bosons under a moderate rotation [16–18]. Under an extremely strong magnetic field or a rapid rotation limit, the centrifugal force tends to reduce the harmonic confinement potential. Then the system will lose its stability, and Eq. (2) reduces to the Landau level. In this paper, we investigate thermodynamics of fermions under a moderate rotation with eigenvalue Eq. (2) and eigenfunction Eq. (3).

In experiments, the thermal energy $k_B T$ far exceeds the level spacing $\hbar\omega$, thus we can replace the sums over all quantum states by an direct integral which is called semi-classical approximation [16]. We take the particle number as an example:

$$N = \sum_{nkm} n(\bar{\varepsilon}) = \sum_{nkm}\left\{\exp\left[(\bar{\varepsilon} - \bar{\mu})/\bar{T}\right] + 1\right\}^{-1} \simeq \int_0^\infty dn \int_0^\infty dk \int_{-\infty}^\infty dm \left\{\exp\left[(\bar{\varepsilon} - \bar{\mu})/\bar{T}\right] + 1\right\}^{-1} = \frac{\bar{T}^3}{\alpha^2} f_3(u). \quad (6)$$

$n(\bar{\varepsilon})$ is Fermi distribution function and $u = \exp(\bar{\mu}/\bar{T})$ is the fugacity. $\bar{T} = k_B T/(\hbar\omega_z)$ and $\bar{\mu} = \mu/(\hbar\omega_z)$ are the dimensionless temperature and chemical potential, respectively. In the process of derivation, we have used the Fermi integration $f_s(u) = (1/\Gamma(s))\int_0^\infty dx\, x^{s-1}/(u^{-1}e^x + 1)$ and Gamma function $\Gamma(s) = \int_0^\infty y^{s-1}e^{-y}\,dy$.

Similarly, the per-particle internal energy and specific heat then take the forms

$$\frac{E}{N} = 3k_B T\frac{f_4(u)}{f_3(u)}, \quad (7)$$

and

$$\frac{C}{N} = \frac{1}{N}\frac{\partial E}{\partial T} = k_B\left[12\frac{f_4(u)}{f_3(u)} - 9\frac{f_3(u)}{f_2(u)}\right]. \quad (8)$$

Eq. (8) is derived from Eq. (7) by using the expressions $f_{s-1}(x) = x\,(d/dx)f_s(x)$ and $\partial N/\partial T = 0$.

Next, we discuss two limit cases of low and high temperatures. In the region of low temperature, $u \longrightarrow \infty$, Fermi integration can be written as a quickly convergent series by Sommerfeld lemma [19,20]

$$f_s(u) = \frac{(\ln u)^s}{\Gamma(s+1)}\left[1 + s\left(s - 1\right)\frac{\pi^2}{6}\frac{1}{(\ln u)^2} + s\left(s - 1\right)\left(s - 2\right)\left(s - 3\right)\frac{7\pi^4}{360}\frac{1}{(\ln u)^4} + \ldots\right]. \quad (9)$$

Then we obtain the Fermi energy which is defined as the chemical potential in absolute zero temperature with Eq. (6):

$$\mu(T = 0) = E_F = \left(6N\hbar^3\omega_0^2\omega_z\right)^{1/3}. \quad (10)$$

The chemical potential and per-particle specific heat are written as

$$\mu = E_F\left[1 - \frac{\pi^2}{3}\left(\frac{k_B T}{E_F}\right)^2\right], \quad (11)$$

and

$$\frac{C}{N} = \frac{\pi^2 k_B^2 T}{3E_F}. \quad (12)$$

In the region of high temperature, $u \longrightarrow 0$, Fermi integration can be expanded as

$$f_s(z) = -\sum_{j=1}^{\infty}\frac{(-z)^j}{j^s}. \quad (13)$$

Consequently, the chemical potential and per-particle specific heat are expressed as

$$\mu = -k_B T \ln\left[6\left(\frac{k_B T}{E_F}\right)^3\right], \quad (14)$$

and