Physica B 481 (2016) 53–58

Contents lists available at ScienceDirect

Physica B

journal homepage: www.elsevier.com/locate/physb

$\mathbb{Z}_2$ slave-spin theory of a strongly correlated Chern insulator

Diana Prychynenko [a,b], Sebastian D. Huber [a,*]

[a] Institut für Theoretische Physik, ETH Zurich, CH-8093 Zürich, Switzerland
[b] Institut für Theoretische Physik, Freie Universität Berlin, Arnimallee 14, 14195 Berlin, Germany

ARTICLE INFO

Article history:
Received 9 September 2015
Received in revised form
22 October 2015
Accepted 26 October 2015
Available online 26 October 2015

Keywords:
Strong correlations
Topology
Slave particles

ABSTRACT

We calculate the phase diagram of the topological honeycomb model in the presence of strong interactions. We concentrate on half filling and employ a $\mathbb{Z}_2$ slave-spin method to find a band insulator with staggered density, a spin-density-wave and a Mott insulating phase. Both the band insulator and the spin-density wave come in various topological varieties. Finally, we calculate the response function relevant for lattice modulation spectroscopy with cold atomic gases in optical lattices.

1. Introduction

The motion of quantum mechanical particles can be associated with interesting topological properties. Beyond the standard example of the quantum Hall effect [1,2], lattice problems with zero net magnetic field attracted considerable recent interest. The honeycomb model with complex next-to-nearest neighbor hopping by Haldane [3] provided the blueprint for a considerable fraction of the current day literature on topological band structures [4,5]. Despite its pivotal role in the development of this field, a direct experimental implementation was only recently demonstrated with ultra-cold atoms [6]. The interesting topological properties of this model arise from the interplay of two energy scales: the strength of the complex next-to-nearest neighbor hopping $t'e^{i\varphi}$, which breaks time-reversal symmetry if $\varphi \notin \{0, \pi\}$, and the sub-lattice potential V, which breaks inversion symmetry. The natural question that poses itself is how an additional energy scale in the form of interactions enriches the picture.

Interactions can alter the physics of particles on topological band structures profoundly. There are several possible scenarios of how interactions can induce new phases. First, for partially filled bands interactions might stabilize gapped quantum liquids akin the Laughlin states for the fractional quantum Hall effects [7–9]. Another possibility is that the interplay of t', V and an interaction scale U leads to symmetry broken states, where the quasi-particles above these states inherit the underlying band-topology [10].

In this paper we discuss how such symmetry broken states can arise at half filling. We explain how they can be described beyond a simple Hartree–Fock theory using slave-particle techniques. Finally, we calculate response functions relevant to current experiments with cold atoms and show how the topological properties of the band structure are revealed. These questions deserve attention as current experiments implement fully tunable honeycomb lattices [11,6] where both the Berry curvature of the bands have been measured [12] and interactions effects have been observed [13].

In the following, Section 2, we introduce the concrete model under investigation. We discuss its possible phases and derive them using both a simple Hartree–Fock (Slater determinant) trial wave function as well as a more sophisticated $\mathbb{Z}_2$ slave-spin method [14,15] which is able to capture interaction effects beyond the physics of Slater determinants. In Section 3 we derive the response functions relevant to the current experiments with ultracold fermions.

2. Ionic Hubbard model at half filling on the honeycomb lattice

2.1. Model

We study the ionic Hubbard model on the honeycomb lattice

$$H = -\sum_{i,j;\sigma} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + \frac{U}{2}\sum_i \left(\sum_{\sigma=\uparrow\downarrow} n_{i\sigma} - 1\right)^2 + \frac{V}{2}\left(\sum_{i\in A,\sigma} n_{i\sigma} - \sum_{i\in B,\sigma} n_{i\sigma}\right). \quad (1)$$

The operators $c_{i\sigma}^\dagger$ create fermions in two different spin species

* Corresponding author.
E-mail address: sebastian.huber@phys.ethz.ch (D. Prychynenko).

http://dx.doi.org/10.1016/j.physb.2015.10.027
0921-4526/

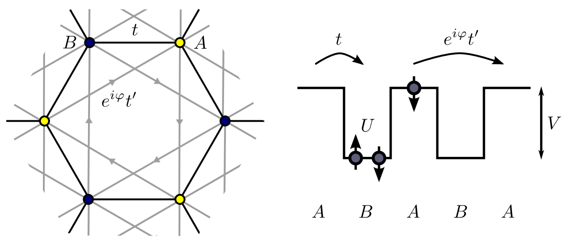

Fig. 1. Setup. (Left) The honeycomb lattice with its two sub-lattices *A* and *B*. The gray arrows indicate the phase convention of the next-to-nearest neighbor hopping, see text. (Right) The different terms in the Hamiltonian: the hopping amplitudes *t* and *t*′; the sub-lattice potential *V*; and the local repulsion between different spin species *U*.

$\sigma = \uparrow, \downarrow$ and t_{ij} denote the hoppings on the honeycomb lattice as indicated in Fig. 1. The hopping to the next-to-nearest neighbors is associated with a phase φ, such that the fermion gains φ, when the hopping is performed clockwise around the unit cell. Finally, we have terms proportional to an onsite repulsion U between the different spin-species and a sub-lattice potential V. We do not specify a chemical potential as we only consider the case of half filling, i.e., one particle per lattice site where the number of $\uparrow$–fermions equals the number of $\downarrow$–fermions.

Let us discuss the well-known phases of this model. For nearest-neighbor hopping only ($t' = 0$) and $V = U = 0$, the half-filled system is a semi-metal. The density of states vanishes linearly at the particle–hole symmetric Dirac points at $\mathbf{K} = (2\pi/a)(2/3, 0)$ and $\mathbf{K}' = (2\pi/a)(1/3, 1/\sqrt{3})$, respectively [16].

Turning on t' breaks the particle–hole symmetry. Moreover, if $\varphi \notin \{0, \pm\pi\}$ the system enters a quantum Hall state with Chern numbers in both spin sectors $C = (C_\uparrow, C_\downarrow) = \pm(1, 1)$ [3]. An inversion-symmetry-breaking term as the sub-lattice potential, $V \neq 0$, opposes the quantum Hall state and eventually renders the system a simple band insulator with strong density modulation [3].

For V=0 but $U > U_{\rm crit}$ the fermions form a spin-density wave (SDW). Note that due to the vanishing density of states at the Dirac point, a finite interaction strength $U_{\rm crit}$ is needed for the SDW to occur [17,18]. Eventually, for $U \gg t$ the fermions get localized in a Mott insulator and form a Heisenberg anti-ferromagnet [19,20].

How are the transitions between these phases characterized? The onset of the SDW goes along with a symmetry breaking of the spin-rotation symmetry SU(2) and can be well described within the Ginzburg–Landau framework. The Mott transition on the other hand is only characterized by a qualitative change in the charge fluctuations, concretely by a vanishing charge imbalance between the two sub-lattices. Finally, the transition where the Chern numbers C are changing requires necessarily a closing of the excitation gap. We are seeking a method that can capture all these phases and transitions in a unified framework.

Readers who are not interested in the technical details can skip the next section and directly advance to Section 2.3.

2.2. Method

In order to describe all aforementioned phases and transitions we employ a slave-spin technique [14,15]. This method, which is tailored to half filling, can track both the excitation spectrum and strongly correlated phases such as the Mott insulator [21,22]. In the following we give a concise account of the slave-spin method and refer the interested reader to Ref. [15] for further details.

The basic building block of the slave-spin method is the introduction of auxiliary degrees of freedom in the form of a constrained slave spin-1/2 (with eigenvalues $I_i^z = \pm 1/2$) on every site

$$c_{i\sigma} = 2I_i^x f_{i\sigma}, \quad I_i^z + \frac{1}{2} = \left(\sum_{\sigma=\uparrow\downarrow} n_{i\sigma} - 1\right)^2, \tag{2}$$

where $f_{i\sigma}$ are regular fermionic operators and $n_{i\sigma} = f_{i\sigma}^\dagger f_{i\sigma}$. The second part of Eq. (2) represents the constraint which slaves the two operators $f_{i\sigma}$ and $\mathbf{I}_i$ to each other. Moreover, it is evident from the constraint that $I_i^z = 1/2$ corresponds to either an *empty or a double occupied* site, while $I_i^z = -1/2$ signals a *singly occupied* site. Expressed in the new operators the Hamiltonian reads

$$H = -\sum_{\langle i,j\rangle,\sigma} 4t_{ij} I_i^x I_j^x f_{i\sigma}^\dagger f_{j\sigma} + \frac{U}{2}\sum_i I_i^z =+ \frac{V}{2}\left(\sum_{i\in A,\sigma} n_{i\sigma} - \sum_{i\in B,\sigma} n_{i\sigma}\right), \tag{3}$$

where we used the constraint to write the interaction part $\propto U$ in the slave-spin sector alone.

Assuming an ansatz for the ground-state wave-function of the form $|\Psi\rangle = |\Psi_f\rangle \otimes |\Psi_I\rangle$ we readily obtain the mean-field Hamiltonian

$$H_{\rm MF} = \langle\Psi_I|H|\Psi_I\rangle + \langle\Psi_f|H|\Psi_f\rangle - \frac{\lambda}{2}\sum_i\left(I_z - 2n_{i\uparrow}n_{i\downarrow} + \sum_\sigma n_{i\sigma}\right), \tag{4}$$

where we added a global Lagrange-multiplier λ to enforce the constraint on average. The resulting meanfield Hamiltonians are given by

$$H_f = \sum_{ij} t_{ij} g_{ij} f_{i\sigma}^\dagger f_{j\sigma} + \frac{V}{2}\left(\sum_{i\in A,\sigma} n_{i\sigma} - \sum_{i\in B,\sigma} n_{i\sigma}\right) - \frac{\lambda}{2}\sum_{i\sigma} n_{i\sigma} + \lambda\sum_i n_{i\downarrow}n_{i\uparrow} \tag{5}$$

and

$$H_I = \sum_{ij} t_{ij}\chi_{ij} I_i^x I_j^x + \left(\frac{U-\lambda}{2}\right)\sum_i I_i^z \tag{6}$$

The two sectors (fermion and slave-sector) are linked via the self-consistency equations for the two renormalization factors

$$g_{ij} = 4\langle\Psi_I|I_i^x I_j^x|\Psi_I\rangle \quad \text{and} \quad \chi_{ij} = 4\sum_\sigma\left(\langle\Psi_f|f_{i\sigma}^\dagger f_{j\sigma}|\Psi_f\rangle + c.\,c.\right). \tag{7}$$

We are now confronted with the problem of solving the two mean-field Hamiltonians (5) and (6). To this end we employ a molecular field approximation to the transverse field Ising model (6) and a Hartree–Fock approximation to (5). The benefit of using the slave-spin approximation over a direct Hartree–Fock approximation to the original model (1) lies in the fact that the slave-spin method allows the interactions to renormalize the hopping strength via g_{ij} and eventually render the system Mott insulating at $g_{ij} = 0$. [23] Note that, strictly speaking, the method described here does not give rise to an actual Mott insulator but to a rather exotic state dubbed an "orthogonal metal" [22]. However, for all values of $g_{ij} \neq 0$ this peculiarity does not arise and we therefore use the slave-spin method here without further elaborating on the orthogonal metal.

2.2.1. Hartree–Fock

We start with the Hartree–Fock approximation of the fermionic