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Lamb wave band gaps in one-dimensional radial phononic crystal plates with periodic double-sided corrugations

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ABSTRACT

In this paper, we present the theoretical investigation of Lamb wave propagation in one-dimensional radial phononic crystal (RPC) plates with periodic double-sided corrugations. The dispersion relations, the power transmission spectra, and the displacement fields of the eigenmodes are studied by using the finite element method based on two-dimensional axial symmetry models in cylindrical coordinates. Numerical results show that the proposed RPC plates with periodic double-sided corrugations can yield several band gaps with a variable bandwidth for Lamb waves. The formation mechanism of band gaps in the double-sided RPC plates is attributed to the coupling between the Lamb modes and the in-phase and out-phases resonant eigenmodes of the double-sided corrugations. We investigate the evolution of band gaps in the double-sided RPC plates with the corrugation heights on both sides arranged from an asymmetrical distribution to a symmetrical distribution gradually. Significantly, with the introduction of symmetric double-sided corrugations, the antisymmetric Lamb mode is suppressed by the in-phase resonant eigenmodes of the double-sided corrugations, resulting in the disappearance of the lowest band gap. Furthermore, the effects of the geometrical parameters on the band gaps are further explored numerically.

1. Introduction

In the last two decades, the propagation of elastic waves in periodic composite media, known as phononic crystals (PCs), has received great interest because of their unique physical properties and potential applications [1–8]. A volume of work has been conducted on the propagation of bulk and surface waves, where the three spatial dimensions of PCs are generally considered infinite or semi-infinite. For example, Kushwaha et al. [9] and Liu et al. [10] respectively investigated theoretically and experimentally the formation mechanisms of band gaps (BGs) of the PCs in rectangular coordinates, and the results demonstrated that the occurrence of the BGs is attributed to two different mechanisms, namely Bragg scattering and localized resonances. Torrent et al. [11–14] and Xu et al. [15] studied the propagation of bulk wave of the radial phononic crystals (RPCs) in cylindrical coordinates and showed that sound propagation was allowed only for certain frequency bands.

Rather than the bulk PCs, the propagation characteristics of Lamb wave in phononic crystal plates had been paid increasing attention for their potential applications in filters, resonators, and waveguides [16–20]. Khelif et al. [21] investigated the propagation of acoustic waves in a phononic crystal slab consisting of piezoelectric inclusions placed periodically in an isotropic host material. It was found that a key parameter for the existence and the width of these complete band gaps was the ratio of the slab thickness to the lattice period. Hsu et al. [22] studied the propagation of Lamb waves in two-dimensional (2D) phononic crystal plates based on Mindlin's plate theory and the plane wave expansion method, and showed that the existence of frequency stop bands was sensitive to the variation of the thickness of the plate. Vasseur et al. [23] calculated elastic band structures of 2D phononic crystal plates by introducing a supercell plane wave expansion method and concluded that the largest absolute forbidden bands occur in the band structure of the phononic crystal plate provided the thickness of the plate is of the order of magnitude of the periodicity of the array of inclusions. Pennec et al. [24] investigated theoretically the band structure of a phononic crystal of finite thickness constituted of a periodical array of cylindrical dots deposited on a thin plate of a homogeneous material and obtained a low-frequency gap, as compared to the acoustic wavelengths in the constituent materials, similarly to the case of locally resonant structures. Wu et al.

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[25] numerically and experimentally analyzed the complete band gaps and resonances in a plate with a periodic stubbed surface. Numerical results show that a complete band gap forms as the stub height reaches about three times the plate thickness. Mourad et al. [26] reported on experimental evidence of the existence of a locally resonant sonic band gap in a 2D stubbed plate. Assouar et al. [27] introduced the concept of double-sided stubbed phononic plate and reported on the theoretical analysis of the enlargement of locally resonant acoustic band gap. Numerical results show that the enlargement of the relative bandwidth of the complete band gap is mainly due to the strong coupling between the same nature of resonant eigenmodes of stubs located in each plate side and the Lamb modes. Wang et al. [28] presented the propagation of Lamb waves in two-dimensional sonic crystals based on a double-sided stubbed plate. As the double stubs in a unit cell arranged more symmetrically on both sides, band width shifts, new band gaps appear, and the bands become flat due to localized resonant modes which couple with plate modes. Significantly, all above mentioned work is mainly concentrated on the propagation of Lamb wave in the PC plates in rectangular coordinates; the propagation characteristics of Lamb wave in the RPC plates has seldom been reported.

Recently, Li et al. [29] firstly considered the propagation characteristics of Lamb wave in one-dimensional RPC plates with periodic single-sided corrugations by using the finite element method based on two-dimensional axial symmetry models in cylindrical coordinates. Numerical results show that the formation mechanism of opening the acoustic band gaps was attributed to the coupling between the Lamb modes and the corrugation mode. In this paper, we theoretically investigate Lamb wave band gaps in one-dimensional RPC plates with periodic double-sided corrugations by using the finite element method based on two-dimensional axial symmetry models in cylindrical coordinates. Numerical results show that the proposed RPC plates with periodic double-sided corrugations can yield several band gaps with a variable bandwidth for Lamb waves. The formation mechanism of band gaps in the double-sided RPC plates is attributed to the coupling between the Lamb modes and the in-phase and out-phases resonant eigenmodes of the double-sided corrugations. We investigate the evolution of band gaps in the double-sided RPC plates with the corrugation heights on both sides arranged from an asymmetrical distribution to a symmetrical distribution gradually. Furthermore, the effects of the geometrical parameters on the band gaps are further explored numerically.

2. Model and methods of calculation

The double-sided RPC plate structure considered here is composed of periodic corrugations deposited on both sides of a homogeneous thin plate, as shown in Fig. 1(a). Fig. 1(b) illustrates the unit cell of two-dimensional axial symmetry model in cylindrical coordinates. The z axis is perpendicular to the homogeneous thin plate. In the unit cell, the lattice constant and plate thickness are defined by a and e respectively, the height and width of the double-sided corrugations are denoted by h_1, h_2 and d respectively. The infinite system of the RPC plate is formed by repeating the unit cell of the two-dimensional axial symmetry model periodically along the r-directions and rotating the axial symmetry model about z-axial to recover the three-dimensional RPC plate model, as shown in Fig. 1(c).

In order to theoretically investigate Lamb wave band gaps in the proposed RPC structures, a series of calculations of dispersion relations and transmission spectra are conducted with the finite element method based on the two-dimensional axial symmetry in cylindrical coordinates [29]. For the calculation of the dispersion relations, the considered structure refers to an infinite system. The governing field equations for elastic wave propagation in cylindrical coordinates are given by

$$\rho\frac{\partial^2 u}{\partial t^2} = (\lambda + 2\mu)\frac{\partial\theta t}{\partial r} - \frac{2\mu}{r}\frac{\partial w'z}{\partial\theta} + 2\mu\frac{\partial w'\theta}{\partial z}$$

$$\rho\frac{\partial^2 v}{\partial t^2} = (\lambda + 2\mu)\frac{\partial\theta t}{r\partial\theta} - 2\mu\frac{\partial w'z}{\partial z} + 2\mu\frac{\partial w'r}{\partial r}$$

$$\rho\frac{\partial^2 w}{\partial t^2} = (\lambda + 2\mu)\frac{\partial\theta t}{\partial z} - \frac{2\mu}{r}\frac{\partial}{\partial r}(rw'_\theta) + \frac{2\mu}{r}\frac{\partial\omega'r}{\partial\theta}$$

where u, v and w are the displacement, ρ is the mass density, t is the time, λ and μ are the Lame constants, r, θ and z represent the coordinate variables in cylindrical coordinates respectively. Additionally, the bulk strain and rotational components are defined as

$$\theta_t = \frac{1}{r}\frac{\partial(ru)}{\partial r} + \frac{1}{r}\frac{\partial v}{\partial\theta} + \frac{\partial w}{\partial z},\ w'_r = \frac{1}{2}\left(\frac{1}{r}\frac{\partial w}{\partial\theta} - \frac{\partial v}{\partial z}\right)$$

$$w'_\theta = \frac{1}{2}\left(\frac{\partial u}{\partial\theta} - \frac{\partial w}{\partial r}\right),\ w'_z = \frac{1}{2}\left(\frac{1}{r}\frac{\partial(rv)}{\partial r} - \frac{1}{r}\frac{\partial u}{\partial\theta}\right)$$

Since the infinite system is periodic along the r-directions, only the unit cell shown in Fig. 1(b) needs to be considered. Stress-free boundary conditions are used for free surfaces in the z-direction, while the periodic boundary conditions are applied at the boundaries between the unit cell and its two adjacent cells with the Bloch theorem.

One knows that with a given value of Bloch wave vector k_r, a group of eigenvalues and eigenmodes can be calculated by solving the eigenvalue problem. By varying the value of Bloch wave vector k_r along the boundaries of the irreducible first Brillouin zone and solving the eigenvalue problem generated by the FEM algorithm, the dispersion relations as well as the eigenmodes of the structure can be obtained.

For the transmission spectrum, a finite system must be defined. We consider a finite array structure composed of N units in the r-direction. Stress-free boundary conditions are still used for free

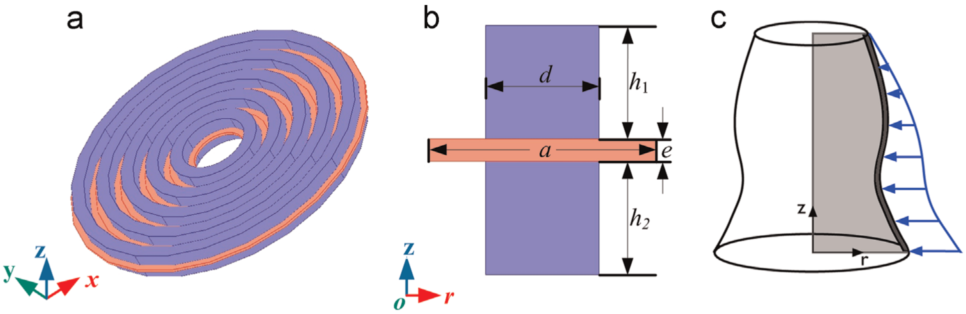

Fig. 1. Schematics of double-sided RPC plates. (a) The three-dimensional model of the proposed double-sided RPC plate; (b) Schematics of the unit cell of the two-dimensional axial symmetry model in cylindrical coordinates; (c) Rotating a two-dimensional geometry to recover a three-dimensional solid.