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Size effects on magnetoelectric response of multiferroic composite with inhomogeneities

Y.M. Yue [a], K.Y. Xu [a,*], T. Chen [a], E.C. Aifantis [b,c,d,e,f]

[a] Shanghai Institute of Applied Mathematics and Mechanics, Shanghai Key Laboratory of Mechanics in Energy Engineering, Department of Mechanics, Shanghai University, Shanghai 200072, China
[b] Laboratory of Mechanics and Materials (LMM), Aristotle University of Thessaloniki, Thessaloniki GR-54124, Greece
[c] Michigan Technological University, Houghton, MI 49931, USA
[d] King Abdulaziz University, Jeddah 21589, Saudi Arabia
[e] School of Mechanics and Engineering, Southwest Jiaotong University, Chengdu, 610031, China
[f] International Laboratory for Modern Functional Materials, ITMO University, St. Petersburg 191002, Russia

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ABSTRACT

This paper investigates the influence of size effects on the magnetoelectric performance of multiferroic composite with inhomogeneities. Based on a simple model of gradient elasticity for multiferroic materials, the governing equations and boundary conditions are obtained from an energy variational principle. The general formulation is applied to consider an anti-plane problem of multiferroic composites with inhomogeneities. This problem is solved analytically and the effective magnetoelectric coefficient is obtained. The influence of the internal length (grain size or particle size) on the effective magnetoelectric coefficients of piezoelectric/piezomagnetic nanoscale fibrous composite is numerically evaluated and analyzed. The results suggest that with the increase of the internal length of piezoelectric matrix (PZT and $BaTiO_3$), the magnetoelectric coefficient increases, but the rate of increase is ratcheting downwards. If the internal length of piezoelectric matrix remains unchanged, the magnetoelectric coefficient will decrease with the increase of internal length scale of piezomagnetic nonfiber ($CoFe_2O_3$). In a composite consisiting of a piezomagnetic matrix ($CoFe_2O_3$) reinforced with piezoelectric nanofibers ($BaTiO_3$), an increase of the internal length in the piezomagnetic matrix, results to a decrease of the magnetoelectric coefficient, with the rate of decrease diminishing.

1. Introduction

The development of advanced electromagnetic materials and composites has penetrated every field of modern technology. Owing to the trend of device miniaturization, multifunctional materials with the unusual electroelastic and/or magnetoelastic properties have drawn increasing interest. For example, magnetoelectric (ME) structures composed of piezoelectric (PE) and piezomagnetic (PM) materials possess interesting magneto-electro-elastic coupling properties, enabling to convert energy from one form to another. Although the fabrication of piezoelectric and piezomagnetic composites with ME coefficients two orders larger than that of single phase multiferroic has been accomplished [1], the desired ME effect are far from sufficient for practical application. In an effort to achieve larger ME effects, numerous researchers have investigated ME couplings in the PE–PM composites both theoretically [2–4] and experimentally [5]. In general, most of the theoretical research is based on macroscopic theories for ME materials and composites.

However, such type of coupling effects may not always be described by classical coupled electro-magneto-elasticity theories when the size is reduced down to the dimensions used in sensors, memory devices and smart structures [6–9]. It heavily depends on the interplay between "external" and "internal" length scales as they defined by specimen dimensions and microstructural size, respectively. Such size-dependent phenomena in piezoelectric materials were found in many experiments [10–12], whereas magnetic properties of piezomagnetic materials are closely related to particle or grain size [13,14]. For PE/PM materials, for example, experiments [15] have demonstrated that the piezoelectric grain size has a significant effect on the ME coefficient. In fact, when the material characteristic dimension is as small as a micron or nano, classical continuum theory should be enhanced with additional nanometers to internal size dependence [16].

As already mentioned, a powerful method to consider mechanical effects in a small volume is to incorporate higher order

* Corresponding author.
E-mail address: kyxu@shu.edu.cn (K.Y. Xu).

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strain/ strain gradients and corresponding internal lengths in the standard continuum mechanic formulation, such as the couple stress theory [17], the micromorphic theory [18], the micropolar theory [19] and the strain gradient theory [20]. These high order theories can also be used to describe coupling effects when electric and magnetic field are simultaneously present with a deformation field. For example, Shodja and Ghazisaeidi [21] studied the size effect in piezoelectric materials by using couple stress theory. Cao et al. [22] analyzed the anti-plane problem for a piezoelectric material with a micro-void or micro-inclusion using the micromorphic electromagnetic theory. In the beginning of 1990 s, Aifantis [23,24] introduced a physically-based gradient elasticity theory and involving only one additional material parameter associated with the dominant material's internal length scale. This simplified theory, commonly known as the gradient elasticity (GRADELA) model, has been widely used to analyze size effects observed during elastic deformation and fracture [25]. An extensive review of gradient elasticity has been given recently in Ref. [26].

Motivated by this, we employ here this simplified GRADELA model in conjunction with standardized continuum electromagnetic theory to consider the size-dependent electro-magneto-elastic response of multiferroic composite. The effort is focused on developing insight for enhancing the ME effect [27] in such materials by exploiting higher order couplings. This is explored by connecting the macro ME response to the underlying microstructure through the introduction of the Laplacian of strain in the standard Hooke's law along with a corresponding internal length associated with the grain size in polycrystal or particle size in PE/PM composites. To simplify the formulation, we focus on description of size effects caused by high order strain gradient, whereas the effect of high order gradient in the electric and magnetic field is disregarded. As a result high order couplings and their influence on the overall size-dependent behavior is attributed to strain gradient effects only. Moreover, in contract to other recent generalized continuum mechanics theories along this direction, the present model is simple to implement numerically and evaluate experimentally. At the same, effective properties for the overall behavior of multiferroic composites can easily be evaluated.

The plan of this paper, the general formulation of multiferroic composite with strain gradient effect is provided. The anti-plane problem of multiferroic composite with inhomogeneities is solved and analytical solution is given. This paper is organized as follows: in Section. 2, the energy functional for an elastic multiferroic body considering strain gradient effects is introduced, and the basic equations are obtained. In Section 3, a general solution for a two-dimensional anti-plane problem is obtained. In Section 4, effective ME moduli for multiferroic composites with inhomogeneities are derived. In Section 5, numerical examples and related analyses to depict the role internal length scale on the overall response of multiferroic material considered are shown. Finally, related conclusions are drawn in Section 6.

2. BASIC formulations

Consider an elastic multiferroic body occupying a domain Ω with boundary $\partial\Omega$. The total energy functional I of the body can be written as

$$I\left[u_i, \phi, \psi\right] = \int_\Omega \left[W\left(\varepsilon_{ij}, \ \varepsilon_{ij,k}, \ E_i, H_i\right) - f_i u_i + q\phi\right]\mathrm{d}v - \int_{\partial\Omega}\left(\bar{t}_i u_i + \ \bar{q}_i Du_i + \overline{q}\,\phi\right)\mathrm{d}s \tag{1}$$

where W is the internal energy density function, ε_{ij} the infinitesimal strain, u_i the displacement field, E_i the electric field, H_i the magnetic field, f_i the body force, and q the body free charge density. The last integral contains the external loading terms with $\bar{t}_i$, $\bar{q}_i$ and $\bar{q}$ denoting the traction vector, the double traction vector and the surface charge density, respectively. Du_i is the surface gradient of the displacement component u_i and the surface gradient operator $D \equiv n_l \partial_l$.

The standard geometric relationships are

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right), \ E_i = -\phi_{,i}, \ H_i = -\psi_{,i}, \tag{2}$$

where ϕ and ψ denote respectively the electric potential and magnetic potential.

Based on the variational principle $\delta I = 0$ for any arbitrary choice from the δu_i, $\delta\phi$ and $\delta\psi$, we finally can obtain the equilibrium equations and boundary conditions

$$\begin{aligned}
&\int_{\partial\Omega}\left(\left[\frac{\partial W}{\partial\varepsilon_{ij}} - \left(\frac{\partial W}{\partial\varepsilon_{ij,k}}\right)_{,k}\right]n_j + \left(\frac{\partial W}{\partial\varepsilon_{ij,k}}n_k n_l\right)_{,l} n_j - \left(\frac{\partial W}{\partial\varepsilon_{ij,k}}n_k\right)_{,j} - t_i\right)\delta u_i \mathrm{d}s \\
&\quad + \int_{\partial\Omega}\left(\frac{\partial W}{\partial\varepsilon_{ij,k}}n_k n_j - \bar{q}_i\right)\left(\delta u_i\right)_{,l} n_l \mathrm{d}s - \int_\Omega\left[\left(\frac{\partial W}{\partial\varepsilon_{ij}}\right)_{,j} \right.\\
&\quad \left. - \left(\frac{\partial W}{\partial\varepsilon_{ij,k}}\right)_{,kj} - f_i\right]\delta u_i \mathrm{d}v = 0, \ \int_\Omega\left[-\left(\frac{\partial W}{\partial E_i}\right)_{,i} + q\right]\delta\phi \mathrm{d}v \\
&\quad + \int_\Omega\left[\left(\frac{\partial W}{\partial E_i}\right)n_i - \bar{q}\right]\delta\phi \mathrm{d}s = 0, \\
&\quad \int_\Omega\left[-\left(\frac{\partial W}{\partial H_i}\right)_{,i}\right]\delta\psi \mathrm{d}v + \int_\Omega\left(\frac{\partial W}{\partial H_i}\right)n_i \delta\psi \mathrm{d}s = 0.
\end{aligned} \tag{3}$$

For the linear case, the internal energy density function W in Eq. (1) can be expanded into the following simplified form

$$\begin{aligned}
W\left(\varepsilon_{ij}, \ \varepsilon_{ij,k}, \ E_i, H_i\right) = {}& \frac{1}{2}\varepsilon_{ij} c_{ijkl}\varepsilon_{kl} - \varepsilon_{ij} e_{ijk} E_k - \varepsilon_{ij} q_{ijk} H_k - \frac{1}{2}E_i \kappa_{ij} E_j \\
& - \frac{1}{2}H_i \mu_{ij} H_j - E_i \lambda_{ij} H_j + \frac{1}{2}\varepsilon_{ij,m} B_{ijmkln}\varepsilon_{kl,n},
\end{aligned} \tag{4}$$

where c_{ijkl}, e_{ijk}, q_{ijk}, κ_{ij}, μ_{ij} and λ_{ij} denote elastic, piezoelectric, piezomagnetic, dielectric permittivity, magnetic permittivity and magnetoelectric moduli, respectively. The coefficient B_{ijmkln} is an additional material constant measuring the effect of strain gradient. Actually, the most general expression of W should include the terms $\varepsilon_{ij,k}\gamma_{ijklm}\varepsilon_{lm}$, $\varepsilon_{ij,k}\chi_{ijkl}E_l$ and $\varepsilon_{ij,k}\eta_{ijkl}H_l$. However, this increases the number of unknown material parameters considerable, and makes the problem more complex and difficult to solve. Therefore, according to the approach of strain gradient elasticity theory [28], we adopt the simplest form possible to analyze the strain gradient effect in multiferroic materials. Higher-order couplings between strain, electric field and magnetic fields may be considered in the further work, if the present results can justify such considerations in conjunction with related experiment results.

From the Eq. (4), the major variables: the elastic stress tensor σ_{ij}^e, double stress tensor μ_{ijk}, electric displacement tensor D_i and magnetic flux tensor B_i are defined through the following relations: