Physica B 478 (2015) 84–88

Contents lists available at ScienceDirect

Physica B

journal homepage: www.elsevier.com/locate/physb

Disorder enhanced conductance in graphene

Yi-Xiang Wang*, Ya-Min Wu

School of Science, Jiangnan University, Wuxi 214122, China

ARTICLE INFO

Article history:
Received 2 June 2015
Received in revised form
9 August 2015
Accepted 25 August 2015
Available online 28 August 2015

Keywords:
Disorder
Transport property
Graphene

ABSTRACT

We study the effect of short-range disorder on the localization property of the electronic state in zigzag graphene, via the calculation of the two-terminal dc conductance with the transfer matrix method. When the disorder is weak, the electron states are localized. However, when the disorder crosses the critical strength, the conductance will be enhanced and may be even quantized as e^2/h at the specific disorder strength. Our numerical calculations suggest that the quantized conductance shows certain robustness to the system size, shape and the Fermi energy. We demonstrate the unconventional behavior from the localization length and the density of states and attribute it to the existence of edge states. The implications of our results are discussed.

1. Introduction

Disorder plays an important role in determining the transport property in two-dimensional (2D) electron gas, and is responsible for striking phenomena such as the metal–insulator transition [1]. A milestone in condensed matter physics is the one-parameter scaling theory, which demonstrates that in a non-interacting 2D electron system, the arbitrarily weak disorder can drive them to be insulator [2]. However, when the external magnetic field or the spin–orbit coupling exists, the scaling theory seems to be invalid. The application of the magnetic field to 2D electron gas breaks the time-reversal symmetry (TRS) of the system and can create the dissipationless edge state, leading to the remarkable phenomenon of the integer quantum Hall effect (QHE) [3]. While the spin–orbit coupling plays a crucial role in driving the quantum spin Hall effect (QSHE) [4,5]. In these two cases, the edge states are robust to the disorder.

Since the discovery of graphene, its fascinating electronic transport property has been the focus of intense experimental and theoretical investigations over the recent years [6–10]. The influence of disorder on the transport property in graphene has also attracted a lot of studies [11–17], but no consensus has achieved yet [18]. In Ref. [13], it was argued that the transport properties in graphene are dominated by diffusion due to disorder. In Ref. [17], the authors suggest when the nondiagonal hopping integral disorder exists, the electronic states at the Dirac points will be delocalized. The recent studies show that disorder can lead to the increase of conductance due to the competition between the increase of carrier density and the simultaneous increase of scattering events between the electrons [19,20].

The theoretical treatment of disorder in graphene requires to introduce the specific model of disorder, such as the short-range and the long-range disorder. The short-range disorder can cause the inter-valley scattering and lead to the Anderson localization [17,21], while the long-range disorder only induces the intra-valley scattering and preserves the relativistic nature of electrons, which will induce the anti-localization behavior [22,23]. However, there are less studies about the electron transport when both the disorder and edge states exist in the system [24].

In this work, we try to study the influence of short-range disorder on the two-terminal dc conductance in zigzag graphene which owns the edge states with the powerful transfer matrix method. We will focus our study on the diffusive regime, which is defined in systems with short-range disorder when the localization length (LL) is larger than the size of the system [25]. We find that when the disorder is weak, the electron states are localized which is consistent with previous studies. Surprisingly, when the disorder is further increased and crossing the critical point, the conductance will be enhanced by disorder and even quantized as e^2/h at the specific disorder strength. The quantized conductance shows certain robustness to the system size, shape and the low Fermi energy. We demonstrate this by calculating the LL and the density of states (DOS) and attribute such unconventional behavior to the fact that more bulk states than edge state participate in the electron transport as the disorder increases. These results remind us of the numerically found nontrivial phase of the topological Anderson insulator (TAI) in HgTe/CdTe quantum well [26–28] which is also driven by the short-range disorder.

* Corresponding author.
E-mail address: wangyixiang@jiangnan.edu.cn (Y.-X. Wang).

http://dx.doi.org/10.1016/j.physb.2015.08.045
0921-4526/

2. Model and method

We start from the honeycomb lattice model with the nearest-neighboring (NN) hopping and short-range disorder simulated by the on-site potential. The tight-binding Hamiltonian is described as follows:

$$H = -\sum_{\langle i,j\rangle} t c_i^{+} c_j + \sum_i U_i c_i^{+} c_i, \tag{1}$$

where c_i^{+} and c_i are the creation and annihilation operators for spinless electrons, respectively, $\langle i, j\rangle$ denotes the NN hopping with strength t. The second term represents the onsite disorder potential which is uniformly distributed as $U_i \in [-W/2, W/2]$ where W is the disorder strength. We will set t as the unit of energy. In Fig. 1(b), the ribbon dispersion (L_y=60) for the zigzag boundary is given, where the edge states appear between two Dirac points of $k_x = 2\pi/3$ and $k_x = 4\pi/3$. The edge states are nearly flat in the vicinity of E=0 [29], indicating that they are degenerate for the top and bottom boundaries. The edge states near $k_x = \pi$ are almost localized on the boundaries completely while those near the Dirac points are much more spread into the bulk. Note the edge states here do not share the same features as those in the QSHE, which can be characterized by the Z_2 topological index and is protected from the TRS [4].

As shown in Fig. 1(a), the graphene ribbon is connected to two highly doped leads for the calculation of dc conductance. Due to the zigzag boundary, the column i of honeycomb lattice can be separated into two sub-columns $i\alpha$ and $i\beta$. When the width of the system is L_y, the wave functions $\Psi_{i\alpha}$ and $\Psi_{i\beta}$, respectively, for sub-columns $i\alpha$ and $i\beta$ can be written in the site representation as

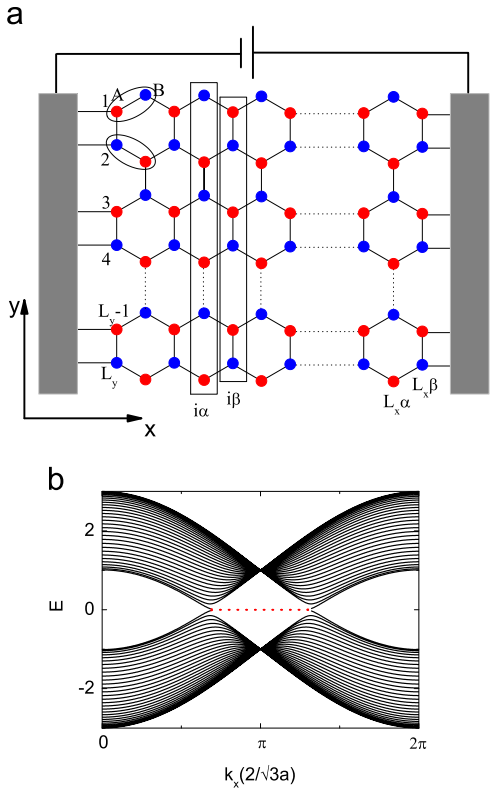

Fig. 1. (a) Schematic plot of the graphene system with zigzag boundary and is connected to two highly doped leads. Column i has been separated as the sub-columns $i\alpha$ and $i\beta$. (b) Plot of the ribbon spectrum. The width is taken as L_y=60. The edge states are shown with dotted (red) lines. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

$$\Psi_{i\alpha} = \begin{pmatrix} \Psi_{i1A} \\ \Psi_{i2B} \\ \vdots \\ \Psi_{iL_y-1A} \\ \Psi_{iL_yB} \end{pmatrix}, \quad \Psi_{i\beta} = \begin{pmatrix} \Psi_{i1B} \\ \Psi_{i2A} \\ \vdots \\ \Psi_{iL_y-1B} \\ \Psi_{iL_yA} \end{pmatrix}, \tag{2}$$

where $\Psi_{ijA(B)}$ is the amplitude of the wave function at sublattice A (B) of j-th row, i-th column. The transfer matrices M_{i1} and M_{i2} of order $2L_y$ connecting neighboring sub-columns are defined as

$$\begin{pmatrix} \Psi_{i+1\alpha} \\ \Psi_{i\beta} \end{pmatrix} = M_{i1} \begin{pmatrix} \Psi_{i\beta} \\ \Psi_{i\alpha} \end{pmatrix}, \quad \begin{pmatrix} \Psi_{i+1\beta} \\ \Psi_{i+1\alpha} \end{pmatrix} = M_{i2} \begin{pmatrix} \Psi_{i+1\alpha} \\ \Psi_{i\beta} \end{pmatrix}. \tag{3}$$

M_{i1} and M_{i2} can be solved from the Schrödinger equation with energy E: $H\Psi_{i\alpha} = E\Psi_{i\alpha}$ and $H\Psi_{i\beta} = E\Psi_{i\beta}$. After a straightforward calculation, the transfer matrix has the following block form:

$$M_{i1} = \begin{pmatrix} N_{i1} & -I_{L_y} \\ I_{L_y} & 0_{L_y} \end{pmatrix}, \quad M_{i2} = \begin{pmatrix} N_{i2} & -I_{L_y} \\ I_{L_y} & 0_{L_y} \end{pmatrix}, \tag{4}$$

where I_{L_y} and 0_{L_y} are respectively the unit matrix and zero matrix with order L_y. The submatrices N_1 and N_2 are

$$N_{i1} = \begin{pmatrix} -\frac{E-U_i}{t} & 0 & 0 & 0 & \cdots \\ 0 & -\frac{E-U_i}{t} & -1 & 0 & \cdots \\ 0 & -1 & -\frac{E-U_i}{t} & 0 & \cdots \\ 0 & 0 & 0 & -\frac{E-U_i}{t} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}, \tag{5}$$

$$N_{i2} = \begin{pmatrix} -\frac{E-U_i}{t} & -1 & 0 & 0 & \cdots \\ -1 & -\frac{E-U_i}{t} & 0 & 0 & \cdots \\ 0 & 0 & -\frac{E-U_i}{t} & -1 & \cdots \\ 0 & 0 & -1 & -\frac{E-U_i}{t} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}. \tag{6}$$

Note that the disordered onsite energies appearing in the diagonal terms are different for different sites.

The transfer matrix connecting the wavefunction of neighboring columns $i+1$ and i is $M_i = M_{i2}M_{i1}$. So the total transfer matrix M which relates the wavefunction of column L_x with the first column is

$$M = \prod_{j=1}^{L_x} M_{L_x-j+1}. \tag{7}$$

For a finite $L_x \times L_y$ quasi-one-dimension system, there are L_y propagating channels along the x direction. The Hermitian matrix $(M^{+}M)^{1/2L_x}$ has $2L_y$ eigenvalues, which should be positive and come in inverse pairs due to the unitarity of the matrix. The L_y positive logarithms of the eigenvalues are called the Lyapunov exponents (LEs) and denoted as $\gamma_1 < \gamma_2 \cdots < \gamma_{L_y}$, which reflect the exponential decay of the corresponding channels [30]. The logarithms of other L_y eigenvalues are the negatives of γ_i. The LL is defined as the reciprocal of the smallest LE $\lambda = 1/\gamma_1$ and plays a crucial rule in the localization theory. In actual numerical calculations, to avoid the terrible overflow in multiplying the transfer matrices, we use the Gram–Schmidt reorthonormalization scheme during the process