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Existence of Majorana fermion mode and Dirac equation in cavity quantum electrodynamics

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ABSTRACT

We present the results of low lying collective mode of coupled optical cavity arrays. We derive the Dirac equation for this system and explain the existence of Majorana fermion mode in the system. We present quite a few analytical relations between the Rabi frequency oscillation and the atom–photon coupling strength to explain the different physical situation of our study and also the condition for massless collective mode in the system. We present several analytical relations between the Dirac spinor field, order and disorder operators for our systems. We also show that the Luttinger liquid physics is one of the intrinsic concepts in our system.

1. Introduction

The recent experimental success in engineering strong interaction between the photons and atoms in high quality microcavities opens up the possibility to use light matter system as quantum simulators for many body physics. Many interesting results are coming out to understand the complicated quantum many body system [1–3].

In the present study one of our goal is to predict the presence of Majorana fermions in our model system. Before we proceed further , we would like to describe very briefly about the appearance of Majorana fermions in quantum condensed matter system. Majorana had introduced a special kind of fermions which are their own antiparticle, i.e., the neutral particle [4,5]. He had introduced this particle to describe neutrions. In recent years, there are several candidates of Majorana fermions in quantum condensed matter system like quantum Hall system with filling fraction 5/2 [6,7]. Kitaev at first found the existence of Majorana fermion mode in one dimensional model [8]. Many research group have already been proposed the physically existence of MFs at the edge state of 1D system like electrostatic defects lines in superconductor, quasi-one dimensional superconductor and cold atom trapped in one dimension [9,10].

In this research paper, we present an extensive derivation of Dirac equation and also the existence of Majorana fermions mode in an optical cavity array. We also present the analytical relation between the Rabi frequency oscillation and the atom–photon coupling strengths to mimic the transverse Ising model, Dirac equation, magnetic ordered state, quantum paramagnetic state and massless excitation. Quantum state engineering of the optical cavity array system is in state-of-the-art due to the rapid technical development of this field [1–3] therefore one can achieve these quantum phases in the laboratory.

2. The model Hamiltonian and Majorana fermion modes

The Hamiltonian of our present study consists of three parts:

$$H = H_A + H_C + H_{AC} \tag{1}$$

The Hamiltonians are the following:

$$H_A = \sum_{j=1}^{N} \omega_e |e_j\rangle\langle e_j| + \omega_{ab} |b_j\rangle\langle b_j| \tag{2}$$

where j is the cavity index. ω_{ab} and ω_e are the energies of the state $|b\rangle$ and the excited state respectively. The energy level of state $|a\rangle$ is set as zero. $|a\rangle$ and $|b\rangle$ are the two stable state of an atom in the

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cavity and $|e\rangle$ is the excited state of that atom in the same cavity. The following Hamiltonian describes the photons in the cavity

$$H_C = \omega_C \sum_{j=1}^{N} a_j^{\dagger} a_j + J_C \sum_{j=1}^{N} (a_j^{\dagger} a_{j+1} + h.\,c.\,), \tag{3}$$

where $a_j^{\dagger}(a_j)$ is the photon creation (annihilation) operator for the photon field in the jth cavity, ω_C is the energy of photons and J_C is the tunneling rate of photons between neighboring cavities. The interaction between the atoms and the photons and also by the driving lasers are described by

$$H_{AC} = \sum_{j=1}^{N} \left[\left(\frac{\Omega_a}{2} e^{-i\omega_a t} + g_a a_j \right) |e_j\rangle\langle a_j| + h.\,c. \right] + [a \leftrightarrow b]. \tag{4}$$

Here g_a and g_b are the couplings of the cavity mode for the transition from the energy states $|a\rangle$ and $|b\rangle$ to the excited state. Ω_a and Ω_b are the Rabi frequencies of the lasers with frequencies ω_a and ω_b respectively.

The authors of Refs. [11–13] have derived an effective spin model by considering the following physical processes: a virtual process regarding emission and absorption of photons between the two stable states of neighboring cavity yields the resulting effective Hamiltonian as

$$H_{xy} = \sum_{j=1}^{N} B\sigma_j^z + \sum_{j=1}^{N} \left(\frac{J_1}{2} \sigma_j^{\dagger} \sigma_{j+1}^{-} + \frac{J_2}{2} \sigma_j^{-} \sigma_{j+1}^{-} + h.\,c. \right) \tag{5}$$

When J_2 is real then this Hamiltonian reduces to the XY model. Where $\sigma_j^z = |b_j\rangle\langle b_j| - |a_j\rangle\langle a_j|$, $\sigma_j^+ = |b_j\rangle\langle a_j|$, $\sigma_j^- = |a_j\rangle\langle b_j|$.

$$H_{xy} = \sum_{i=1}^{N} (B\sigma_i^z + J_1(\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y)$$

$$+ J_2(\sigma_i^x \sigma_{i+1}^x - \sigma_i^y \sigma_{i+1}^y)) = \sum_{i=1}^{N} B(\sigma_i^z + J_x \sigma_i^x \sigma_{i+1}^x + J_y \sigma_i^y \sigma_{i+1}^y). \tag{6}$$

With $J_x = (J_1 + J_2)$ and $J_y = (J_1 - J_2)$.

$$J_1 = \frac{\gamma_2}{4} \left(\frac{|\Omega_a|^2 g_b^2}{\Delta_a^2} + \frac{|\Omega_b|^2 g_a^2}{\Delta_b^2} \right), \quad J_2 = \frac{\gamma_2}{2} \left(\frac{\Omega_a \Omega_b g_a g_b}{\Delta_a \Delta_b} \right). \tag{7}$$

$$B = \frac{\delta_1}{2} - \beta \tag{8}$$

The analytical expression for $\delta_1, \beta, \gamma_1, \gamma_2$, Δ_a and Δ_b is given in Ref. [14]. The system reduces to the Ising model with transverse field at $J_1 = J_2$, i.e., J_x becomes $J_1 + J_2$ and $J_y = 0$. The effective Hamiltonian becomes the transverse Ising model which is studied in the previous literature [15–17]. Here our main motivation is to use some of important results of this model Hamiltonian to discuss the relevant physics of array of cavity QED system.

Before we proceed further, we would like to discuss in detail the analytical relation between the different coupling constants of cavity QED system to achieve this Hamiltonian. In the microcavity array, the condition for $J_1 = J_2$ achieves when

$$\Omega_a^2 g_b^2 \Delta_b^2 + \Omega_b^2 g_a^2 \Delta_a^2 = 2\Omega_a \Omega_b g_a g_b \Delta_a \Delta_b. \tag{9}$$

The above condition implies that $\Omega_a = \Omega_b g_a \Delta_a / g_b \Delta_b$. The only constraint is that $\Delta_a \neq \Delta_b$, the magnetic field diverges when $\Delta_a = \Delta_b$. At the same time, $\Omega_a = \Omega_b$ and $g_a = g_b$ are also not possible because this limit also leads to the condition $\Delta_a = \Delta_b$. Suppose we consider, $\Omega_a = \alpha_1 \Omega_b$, $g_a = \alpha_2 g_b$ and $\Delta_a = \alpha_3 \Delta_b$. These relations imply that $\alpha_1^2 + \alpha_2^2 \alpha_3^2 = 2\alpha_1 \alpha_2 \alpha_3$. $\alpha_1 = \alpha_2 \alpha_3$. α_1, α_2 and α_3 are the numbers. These analytical relations help to implement the transverse Ising model Hamiltonian but α_1, α_2 and α_3 should not be equal to 1.

The quantum state engineering of cavity QED is in state-of-the-art due to the rapid progress of technological development of this field [1]. Therefore one can achieve this limit to get the desire quantum state.

$$H_T = B \sum_{j=1}^{N} (\sigma_z(j) + \lambda \sigma_x(j) \sigma_x(j+1)), \tag{10}$$

where $\lambda = (J_1 + J_2)/B$. The transverse Ising model was studied widely in the literature and also exhibits a quantum phase transition between the magnetically ordered state to the quantum paramagnetic phase for $\lambda > 1$ and $\lambda < 1$ respectively [15–17].

Now we express the condition for the magnetic order phase and quantum paramagnetic phase in terms of the physical parameters of the optical cavity QED system which gives us the relevant physics of the system.

The condition for the magnetic ordered system can be expressed as

$$\frac{\gamma_2}{4} \left(\frac{\Omega_a g_b^2}{\Delta_a^2} + \frac{\Omega_b g_a^2}{\Delta_b^2} \right) + \frac{\gamma_2}{2} \left(\frac{\Omega_a \Omega_b g_a g_b}{\Delta_a \Delta_b} \right) > \omega_{ab} - \frac{\omega_a - \omega_b}{2} - 2\beta. \tag{11}$$

The condition for the quantum paramagnetic phase is

$$\frac{\gamma_2}{4} \left(\frac{\Omega_a g_b^2}{\Delta_a^2} + \frac{\Omega_b g_a^2}{\Delta_b^2} \right) + \frac{\gamma_2}{2} \left(\frac{\Omega_a \Omega_b g_a g_b}{\Delta_a \Delta_b} \right) < \omega_{ab} - \frac{\omega_a - \omega_b}{2} - 2\beta. \tag{12}$$

When the applied magnetic field is absent, the effective Ising model has two degenerate ground states. The ground states are $|A\rangle = \Pi_j | \rightarrow \rangle_j$, $|B\rangle = \Pi_j | \leftarrow \rangle_j$. For a finite magnetic field but less than $J_1 + J_2$, the system has a tendency to flip the pseudo-spin. At that phase one can write down the true eigen state, $|\psi_A\rangle = 1/\sqrt{2}\,(|A\rangle + |B\rangle)$, $|\psi_B\rangle = 1/\sqrt{2}\,(|A\rangle - |B\rangle)$. Now our main intention is to recast this spin model in spinless fermion model through the Jordon–Wigner transformation which relates the spin operators to the spinless fermion operators. We use the following relation:

$$\sigma_z = 2c^{\dagger}(j)c(j) - 1, \quad \sigma_x(j)\sigma_x(j+1)$$
$$= (c^{\dagger}(n) - c(n))(c^{\dagger}(n+1) - c(n+1)).$$

One can write the Hamiltonian after the Jordon–Wigner transformation as

$$H = 2 \sum_{j=1}^{N} c^{\dagger}(j)c(j) + \lambda (c^{\dagger}(j) - c(j))(c^{\dagger}(j+1) - c(j+1)) \tag{13}$$

We solve this Hamiltonian, to get the energy spectrum by taking the Fourier transform.

$$c(j) = \frac{1}{\sqrt{N}} \sum_k c_k e^{-ika}, \quad c^{\dagger}(j) = \frac{1}{\sqrt{N}} \sum_k c_k e^{ika}.$$

where c_k and $c_k^{\dagger}$ are the fermionic annihilation and creation operator in momentum space.

The Hamiltonian reduce to

$$H = 2 \sum_{k>0} (1 + \lambda \cos k)(c_k^{\dagger} c_k + c_{-k}^{\dagger} c_{-k})$$

$$+ 2i\lambda \sum_{k>0} \sin k (c_k^{\dagger} c_{-k}^{\dagger} + c_k c_{-k}) \tag{14}$$

Now our main task is to express the Hamiltonian in the diagonalized form. We follow the Bogoliubov transformation.

$$\eta_k = \alpha_k c_k + i\beta_k c_{-k}^{\dagger} \quad \text{and} \quad \eta_{-k} = \alpha_k c_{-k} - i\beta_k c_k^{\dagger}, \quad k > 0.$$

The operator η_k and $\eta_k^{\dagger}$ are the fermionic operators. We use the following relations: