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Fractional quantum Hall effect revisited

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ABSTRACT

The topology-based explanation of the fractional quantum Hall effect (FQHE) is summarized. The cyclotron braid subgroups crucial for this approach are introduced in order to identify the origin of the Laughlin correlations in 2D (two-dimensional) Hall systems. Flux-tubes and vortices for composite fermions in their standard constructions are explained in terms of cyclotron braids. The derivation of the hierarchy of the FQHE is proposed by mapping onto the integer effect within the topology-based approach. The experimental observations of the FQHE supporting the cyclotron braid picture are reviewed with a special attention paid to recent experiments with a suspended graphene. The triggering role of a carrier mobility for organization of the fractional state in Hall configuration is emphasized. The prerequisites for the FQHE are indicated including topological conditions substantially increasing the previously accepted set of physical necessities. The explanation of numerical studies by exact diagonalizations of the fractional Chern insulator states is formulated in terms of the topology condition applied to the Berry field flux quantization. Some new ideas withz regard to the synthetic fractional states in the optical lattices are also formulated.

1. Introduction

Observation of the fractional quantum Hall effect (FQHE) was one of the most important discoveries of the 20th century. The experiment carried out by Tsui et al. revealed plateaus in the longitudinal resistance appearing concomitantly with dips in the transverse one for 2DEG partially filling the lowest Landau level (LLL) upon strong magnetic fields and temperatures below 4 K [1]. An origin of the elder integer quantum Hall effect (IQHE) was explained shortly after its discovery within a single particle approach including topology arguments [2]. Actually, the first explanation was even simpler—it is assumed that for completely filled Landau levels (LLs) an electron cannot scatter between different one particle states and a current cannot flow in the direction of a voltage ($R_{xx} = 0$). In opposition, the fractional quantum Hall effect is a collective phenomenon being a manifestation of strong interparticle correlations and, despite the intensive research, its nature is still not fully understood. The basic prerequisite for the FQHE formation is the flat band with quenched kinetic energy, as in the almost degenerated LLL in the presence of interaction (and massively degenerated without interactions). Reducing of the kinetic energy role allows for the subtle interaction effects resulting in the organization of correlated multiparticle states. An important role is played by a very special 2D topology—there is no evidence of the FQHE in three-dimensional (3D) samples.

The first step towards the description of correlations in the LLL was taken by Laughlin. He proposed a wave function for $\frac{1}{q}$ (q-odd) filling factors formed with a Jastrow polynomial and a Gaussian factor [2]

$$\Psi_L(z_1, \ldots z_N) = \prod_{i,j=1, i>j}^{N} (z_i - z_j)^q e^{-\sum_{i=1}^{N} (|z_i|^2/4l^2)}. \tag{1}$$

where $z_i = x_i + iy_i$ is a complex position of ith particle on a plane, $l = \sqrt{\frac{\hbar c}{eB}}$ is a magnetic length. The representation of the Coulomb repulsion in the form of Haldane pseudopotential revealed that this Laughlin function (LF) describes the exact ground state for N charged particles placed on a plane, if one neglects the long-range part of the Coulomb forces [4–6]. Division of the interaction for near- and long-range parts is expressed by its projection onto the relative angular momenta of particle pairs: values greater than $q-2$ correspond to the long-range tail, while values lower than $q-2$—to the near range part of the field. It has already been proved that the long-range tail influences only slightly the exact ground state obtained only with a short-range part included. Note that the LF is actually a generalized Slater function [7] with a p power introduced in the Vandermonde polynomial

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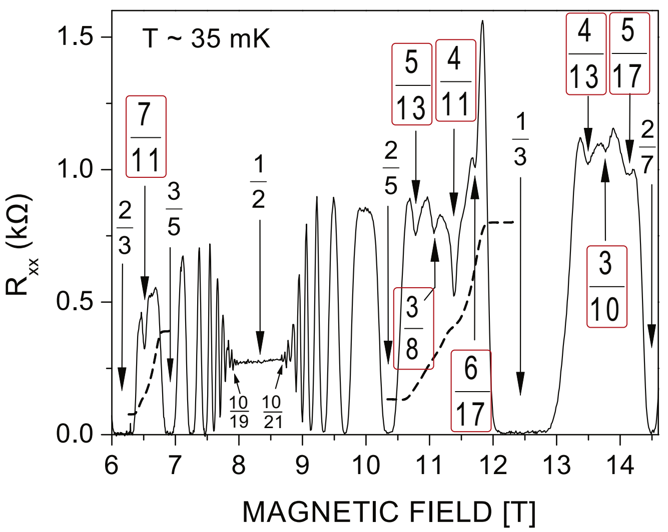

Fig. 1. Observation of the FQHE in a GaAs/AlGaAs quantum well with an electron density of 10^{11} 1/cm^{-2}. R_{xx} for $\frac{2}{3} > \nu > \frac{2}{7}$ at the temperature equal T ~35 mK is presented. The Hall resistance R_{xy} in the region of $\nu = \frac{7}{11}$ and $\nu = \frac{4}{11}$ is marked with a dotted line (after Ref. [3]). Fractions outside the standard CF hierarchy are indicated in color. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

$$\Psi_S(z_1, \ldots z_N) = \prod_{i,j=1,i>j}^{N} (z_i - z_j) e^{-\sum_{i=1}^{N} (|z_i|^2/4l^2)}, \tag{2}$$

The main difference between these two functions is disclosed when two particles are exchanging on a plane—the phase shift obtained by the Laughlin function is p times greater ($p\pi$) than by the Slater function (π). It should be mentioned that exchanges of particles located on locally 2D manifolds are considerably different than those for particles located on manifolds of higher dimensions. In the latter case the exchanges correspond to simple particle permutations and are expressed with algebraic properties of the multi-argument wave function. For 2D spaces the exchanges are related to the full braid group elements, different from simple particle permutations. The full braid group is actually a homotopy group (π_1) of the configuration space of the N indistinguishable particle system, so a group of classes of homotopic trajectories. Finally, association of the algebraic properties of multi-argument functions with exchanges of particles described by these functions may be misleading and exchanges of function arguments must be referred to the braid group distinct than the permutation group (Fig. 2). However, the mentioned difference is considered to be a hallmark of Laughlin correlations, even though the unitary factor $e^{iq\pi} = -1$ equals to $e^{i\pi} = -1$. So, this property does not allow us to distinguish correlated particles from ordinary fermions.

Despite the fact that neglecting the topology and relying only on properties of the LF is insufficient for a complete explanation of the FQHE origin, some quite successful theories where introduced. One of these theories is the so-called composite fermion (CF) model, proposed by Jain [8]. It assumes that these new particles are just electrons dressed with $q-1$ magnetic field flux-quanta (q is the number of flux quanta of an auxiliary magnetic field) resulting from interactions, in analogy to the three-dimensional Landau quasiparticles in solids [9]. The proper phase shift is obtained due to the Aharonov–Bohm effect, since an exchanging composite particle sees the flux quanta placed on the opposite fermion. The success of the CF construction lies in the possibility of reduction of the FQHE of electrons to the well-understood IQHE of composite fermions experiencing a lower effective magnetic field (the localized magnetic flux quanta screen an external field, leading to $q-(q-1)=1$ quanta per composite particle). However, assertion of flux tubes—as a result of the Coulomb repulsion forces—should not be used in a 2D space without proper explanation (the matrix element of the interaction is not a continuous function of distance). As a consequence, this simplistic, one-particle theory seems to model with artificial objects, rather than explain, the real complicated behavior of the particles. Nevertheless, the CF idea allows for estimating the main line of the FQHE hierarchy —$\nu = \left((q-1) \pm \frac{1}{n}\right)^{-1} = \frac{n}{(q-1)n \pm 1}$ (q—odd integer, n—integer) [8]—corresponding to the complete filling of the nth LL in the screened magnetic field. This resultant effective field can be oriented along or opposite to the original one, thus $\pm$ is appearing in the filling factor expression. The compatibility of the hierarchy with the experiment suggests that despite all problems and ambiguities mentioned above, it models some more fundamental properties of 2D systems in strong magnetic fields. For a relatively long time these properties were not recognized. A progress was achieved recently [10] in terms of cyclotron braid subgroups, which will be also presented in more details in the present paper.

The competitive construction of CFs was formulated shortly afterwards by Read [11,12]. This formulation is based on the conception of collective fluid-like objects called vortices, which are characterized with q-vorticity and are similar to the well-known constructions present in superfluid systems. The vortices are pinned to bare fermions—resulting complexes are also called composite particles (not only composite fermions, but also composite bosons), since they reproduce the Laughlin correlations [11]. However, the vortex is expressed with a fragment of the Jastrow polynomial—$V(z) = \prod_{j=1}^{N} (z_j - z)^q$. Thus, all particles contribute to the vortex definition and the vorticity coincides with the q-power in the LF. Therefore, it is not surprising that fermions dressed with vortices (when the argument z is assumed z_i) reproduce the Laughlin correlations, since the vortex notion arises immediately from a simple decomposition of the LF with the vorticity taken from the Jastrow polynomial known in advance. Thus, this competitive conception do not explain the FQHE origin (it can be rather understand as a different representation of the LF) and its significance is rather of illustrative type.

Both types of composite particles, with vortices or with flux tubes, are thus phenomenological in nature, and the question arises as of what is a more fundamental reason of Laughlin correlations in 2D charged systems upon sufficiently strong magnetic field and of how are they linked to specific 2D topology. The role of topology in the strongly correlated state creation was noticed [13–15] in the context of exceptional topological properties of a plane and locally 2D manifolds like sphere or torus. This unique topology of planar systems is linked with an exceptionally rich structure of their braid groups in comparison to ones of higher dimensional spaces (R^d, $d>2$) [16]. As it was already mentioned, the full braid group is a group of multi-particle closed trajectory classes, disjoint and topologically nonequivalent (trajectories from different classes cannot be continuously deformed one into another). In the case of 2D spaces the full braid group is infinite, while for manifolds of higher dimensions it is finite and equal to S_N—the permutation group of N elements [16]. This property makes 2D systems exceptional in geometry–topology sense, which inherently lies in

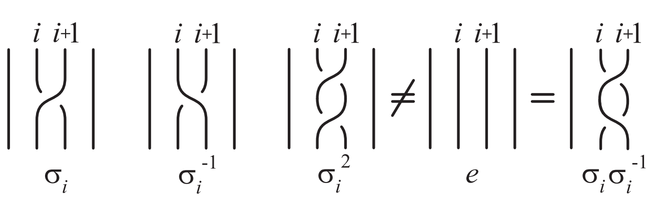

Fig. 2. The geometrical presentation of σ_i—the generator of the full braid group of R^2 space and σ_i^{-1}—its inverse; in 2D $\sigma_i^2 \neq e$.