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Effective temperature of the non-equilibrium electrons in a degenerate semiconductor at low lattice temperature

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ABSTRACT

The energy balance equation for the electron–phonon system is recast taking the degeneracy of the carrier ensemble into account. The effect of degeneracy on the field dependence of the temperature of the non-equilibrium carriers has been studied by solving the same equation. The high field distribution function of the carriers is assumed to be given by the Fermi Dirac function at the field dependent carrier temperature. The distribution function has been approximated in a way that facilitates analytical solution of the problem without any serious loss of accuracy. The field dependence of the electron temperature thus obtained seems to be significantly different from what follows had the degeneracy not been taken into account. The agreement of the results obtained from the present analysis with the available experimental data for Ge and InSb are quite satisfactory. The scope of further refinement of the present theory is highlighted.

1. Introduction

In the presence of a relatively high field the free carriers in a semiconductor may be significantly perturbed from the state of thermodynamic equilibrium with the host lattice atoms. The critical field at which the electrons may be drifted to such a significantly perturbed state in any material increases with the increase of the lattice temperature and with the decrease of the values of the initial mobility. For example, in n-Ge, when the lattice temperature is low, say around 5 K, the electrons may be so perturbed for a field of only a few V cm^{-1}, and in InSb, when the lattice temperature is around 2 K, similar perturbation may be observed for a fraction of a V cm^{-1}. On the other hand, if the lattice temperature is raised to around room temperature, significant perturbation of the carriers requires fields of several kV cm^{-1} in Ge and some hundred V cm^{-1} in InSb. Thus, even a seemingly low field may effectively turn out to be quite high if the lattice temperature is low enough, and this may indeed cause significant perturbation of the carrier system from the state of thermodynamic equilibrium at the low temperatures. Such a perturbed ensemble is known to exhibit a number of novel phenomena which are technologically important from the device point of view. Thus the problem of electrical transport in semiconductors in the presence of an effectively high field at any lattice temperature, low or high has been of interest for many years [1–6].

To make a theoretical analysis of the characteristics of a material in the presence of a relatively high field at any lattice temperature, low or high, one needs to solve the Boltzmann Transport equation taking into account the various interactions of the electrons with the lattice defects. But, to arrive at an analytical solution of the transport equation under such conditions is almost always beset with much mathematical difficulties. So one has to either adopt simplifying assumptions which, very often, may compromise with the physical validity of the results, or go for some exact numerical techniques.

When the carrier concentration is large so that the energy exchange between the carriers is much faster than that between the carriers and the lattice, the carriers share their energy mainly among themselves. The carriers then, in the presence of a relatively high field at any lattice temperature T_L, low or high, may become 'hot' attaining a field dependent effective temperature T_e that exceeds T_L. Obviously T_e gives a measure of the average energy of the ensemble. Under these conditions, the high field energy distribution for a degenerate ensemble may be approximated by the Fermi Dirac distribution at a carrier temperature T_e [1,2]. The mean carrier energy as calculated in the diffusion approximation is given by

$$< \varepsilon >_{F.D.} = k_B T_e \frac{F_{3/2}(\eta_e)}{F_{1/2}(\eta_e)}$$

where k_B is Boltzmann Constant, $\eta_e = \varepsilon_F / k_B T_e$, ε_F is the Fermi

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energy and $F_k(\eta_e)$ are the Fermi integrals [6,7]. However, for relatively lower concentration of the carriers when $\varepsilon_F < 0$ and $|\varepsilon_F|$ is much larger than k_BT_e, the materials seem to be non-degenerate and the Fermi function simplifies to the Maxwellian function. The average energy under this condition simplifies to $3k_BT_e/2$ [6,8]. In either case, the field dependence of the effective temperature of the electrons may be calculated from the solution of the energy balance equation. The solution however, depends upon the dominant type of interaction under the prevailing condition and the band structure of the material. The interaction with the optical and intervalley phonons may be dominant for lattice temperatures above some hundred degrees. On the other hand, the interaction with the intravalley acoustic phonons is intrinsic and may dominate along with the impurity scattering at the lower temperatures. However, the collision with impurities being elastic, the intravalley acoustic phonon scattering will dominate in determining the field dependence of the effective electron temperature in the lower temperature regime. Such field dependence has already been worked out for a non-degenerate material [1,2,6]. The dependence is of the simple form $T_n(T_n - 1) \sim E^2$ where $T_n = T_e/T_L$. Obviously such dependence is predicted for samples having lower carrier concentrations and at the higher lattice temperatures.

At lower temperatures, however, as a result of increasing the doping level the electron concentration in an n-type material increases, and when it eventually exceeds the effective density of states, the Fermi level ε_F moves into the conduction band. Under this condition and when ε_F is not much lower than k_BT_L of the band edge, and the electron densities are beyond the insulator to metal transition, the electron ensemble turns out to be degenerate. A rough estimate of the critical donor concentration N_D for the onset of the degeneracy may be estimated from $\varepsilon_F = \left(\frac{\hbar^2}{2m^*}\right)\left(3\pi^2 N_D\right)^{2/3} > E_d$ where $\hbar = h/2\pi$, h being Planck's constant, m^* is the effective mass of an electron and E_d is the donor ionization energy. The degeneracy is said to be extreme when $\varepsilon_F \gg k_BT_L$. The degenerate materials serve as the basis for a number of useful semiconductor devices [8,9]. The knowledge of the electrical transport characteristics of degenerate materials thus being important, some aspects of such characteristics have already been studied by a number of workers including one of the present authors [10–14].

The purpose of the present communication is to obtain the electric field dependence of the effective electron temperature T_e for a degenerate ensemble of electrons which is subjected to a relatively high field, and under the condition when the interaction with the intravalley acoustic phonons dominate. Before solving the energy balance equation for the electron–phonon system, it is first recast taking into account the degeneracy of the electron ensemble, the energy distribution of which is described by the F.D. statistics at the field dependent effective temperature T_e. Because of the complexity of the F.D. distribution, the integrals that occur while solving the energy balance equation are not usually amenable to analytical evaluation. In the present analysis some alternative model of the distribution has been used so that the integrals can indeed be carried out analytically without compromising with the validity of the final results. The numerical results obtained for Ge and InSb from the present analysis are then compared with other theoretical and experimental results. The agreement of the results from the present analysis with that from the experiments seems to be significantly better. The calculations have been carried out for any finite value of the degeneracy and the lattice temperature.

2. Development

The condition for energy conservation of the electron–phonon system may be expressed as [1,2,6]

$$\int \frac{\partial f\left(\vec{k}\right)}{\partial t}\bigg|_{coll} \varepsilon d\vec{k} = \int \frac{\partial f\left(\vec{k}\right)}{\partial t}\bigg|_{field} \varepsilon d\vec{k} \tag{1}$$

where $\frac{\partial f\left(\vec{k}\right)}{\partial t}$ is the time rate of change of the distribution function $f\left(\vec{k}\right)$, ε is the energy of an electron with wave vector $\vec{k}$. The distribution function in the diffusion approximation may be written as

$$f\left(\vec{k}\right) = f_0(\varepsilon) + k\cos\theta f_1(\varepsilon)$$

θ being the angle between the wave vector $\vec{k}$ and the electric field $\vec{E}$.

Let us consider a volume V of an isotropic, degenerate semiconductor material with a single, parabolic, spherically symmetric conduction band. Taking into account the four processes of absorption and emission involving a phonon of wave vector $\vec{q}$ that leads to the scattering of the electrons into and out of the state $|\vec{k}\rangle$ for transitions to and from the states $|\vec{k} + \vec{q}\rangle$ and $|\vec{k} - \vec{q}\rangle$, it follows from the perturbation theory that

$$\begin{aligned}\frac{\partial f\left(\vec{k}\right)}{\partial t}\bigg|_{coll} = \frac{2\pi}{\hbar} \sum_{\vec{q}} \Bigg[&\left|\left\langle \vec{k}, N_{\vec{q}} \pm 1|H'|\vec{k} \pm \vec{q}\right\rangle\right|^2 \delta\left(\varepsilon_{\vec{k}} - \varepsilon_{\vec{k}\pm\vec{q}} \pm \hbar\omega_{\vec{q}}\right) \\ &f\left(\vec{k} + \vec{q}\right) f\left(\vec{k}\right) - \left|\left\langle \vec{k} \pm \vec{q}, N_{\vec{q}} \pm 1|H'|\vec{k}\right\rangle\right|^2 \\ &\delta\left(\varepsilon_{\vec{k}\mp\vec{q}} - \varepsilon_{\vec{k}} \pm \hbar\omega_{\vec{q}}\right) f\left(\vec{k}\right) f\left(\vec{k} + \vec{q}\right)\Bigg]\end{aligned} \tag{2}$$

where $N_{\vec{q}}$ is the equilibrium distribution function of the phonons, $\omega_{\vec{q}}$ is the angular frequency of a phonon with the wave vector $\vec{q}$. The upper or the lower sign in the first term must be taken for the processes of emission and absorption respectively, whereas those signs in the second term stand for the reverse processes.

It is well known that when the band edge shift is linearly dependent upon the strain, one can neglect the spin exchange scattering [15], as such the matrix element for the transition remains unchanged for a degenerate ensemble, and hence it is given by [1,2,8]

$$\left|\left\langle \vec{k} \pm \vec{q}, N_{\vec{q}} \pm 1|H'|\vec{k}\right\rangle\right|^2 = \frac{E_1^2 \hbar q}{2\rho u_l V}\begin{bmatrix} N_{\vec{q}} \\ N_{\vec{q}} + 1 \end{bmatrix} \tag{3}$$

where E_1 is the deformation potential constant, ρ is the density of the material and u_l is the average acoustic velocity. Now following the standard procedure [1,2] one can obtain for a degenerate ensemble