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On the diffusion-strain coupling and dispersion of surface waves in transversely isotropic laser-excited solids

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ABSTRACT

The present paper is aimed at studying the boundary value problem in elasticity theory concerning the propagation behavior of harmonic waves and vibrations on the surface of the transversely isotropic laser-excited crystalline solids with atomic defect generation. Coupled dynamical diffusion-–deformation interaction model is employed to study this problem. The frequency equations of surface waves in closed form are derived and discussed. The three motions, namely, longitudinal, transverse, and diffusion of the medium are found to be dispersive and coupled with each other due to the defect concentration changes and anisotropic effects. The phase velocity and attenuation coefficient of the surface waves get modified due-to the defect–strain coupling and anisotropic effects, and are also influenced by the defect relaxation time. A softening of frequencies of surface acoustic waves (instability of frequencies) is obtained. Relevant results of previous investigations are deduced as special and limiting cases.

1. Introduction

Theory of elastic waves coupled to defect dynamics is an important generalization of the classical theory of elasticity. Problems concerning defect generation play a vital role in theoretical and practical problems of laser additive micro-and nanotechnologies, materials processing technology and etc. The theory of elasticity concerning the solid elastic material consisting of a distribution of defects is used widely for investigating such phenomena as laser annealing, fast recrystallization, selective laser sintering of powders and laser-assisted thin-film deposition process for which the use of the classical elasticity theory for mechanical behavior of materials is inadequate. All above-mentioned processes can be accompanied by the generation of atomic point defects (interstitial atoms, vacancies, adatoms, electron–hole pairs). Also, efficient generation of nonequilibrium atomic defects may occur as a result of the action of intense impulse external energy fluxes (laser and corpuscular radiations) on condensed media or as a result of mechanical, thermochemical, and electric treatments of materials. The theory to include the effect of defect concentration change, known as diffusion–strain (DS) coupled theory, is well established [1,2]. This theory deals the deformational behavior of materials with a distribution of nonequilibrium atomic defects, where the concentration of defects is included among the kinematic (diffusional) variable. The theory reduces to the classical theory in the limiting case of concentration of defects tending to zero.

DS coupled theory has been receiving a lot of attention for the past two decades. Extensive theoretical efforts have been made so far to study the dynamical interaction between the defect concentration and mechanical fields in laser-excited solids in the context of DS coupled model. The problem of linear elasto-diffusive Rayleigh-type wave propagation on the surface of isotropic semi-infinitive solids and elastic thin plates with atomic defect generation has been considered by [1–5]. It has been shown that the surface waves in these types of media are dispersive in contrast to classical theory of elasticity in which Rayleigh-wave motions are not dispersive at any frequency. In general case, the surface waves are found to exhibit frequency dependent dispersion and are accompanied with attenuation or amplification. In Refs. [6,7], the small-scale effects on propagation of elasto-diffusive waves have been investigated in the context of coupled DS theory.

For high intensities of incident laser radiation, the concentration of defects becomes so high that one should expect the appearance of co-operative effects in an ensemble of interacting (through the self-consistent elastic field of displacements in a medium) defects. The 1D and 2D self-organization of nonlinear coupled periodic and localized strain–defect structures (solitons or solitary waves) due to concentration-elastic instability were considered in [8,9]. A mechanism on the development of the instability is due to the coupling between defect dynamics and elastic field of the solids. Stabilization of this instability is due to the nonlinearity of the elastic continuum. The mathematical models of these studies were based on the wave type (hyperbolic) equations of motion for the displacement vector and diffusion type (parabolic) equation for atomic defect concentration accounting for elastic and concentration nonlinearities.

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The theory of mechanical waves coupled to atomic defect dynamics and including thermal change effects in solids under the action of laser pulses has been considered by Mirzade [10] and Bargmann and Favata [11]. Some features of the physical problems coupling diffusion, mechanics and thermal waves in geometrically linear and nonlinear solids has been studied in Refs. [12–14].

An overview of methods for analysis of elastic fields in solids from various structural defects (dislocation, inhomogeneous inclusions, grain boundaries and cracks) was presented by Mura [15] using the concept of eigenstrains in micromechanics. In the Mura's eigen-strain theory, the eigen-strains corresponding to each defect are conveniently expressed in terms of a defect density tensor. This theory has been applied successfully to modeling many important processes in crystalline solids such as diffusional phase transformations and microstructure coarsening, which involve diffusional redistribution of atoms under the influence of stresses arising from coherent compositional inhomogeneities as well as from structural defects [16,17]. The spatial distribution of defects in these models is described by the space-dependent eigenstrains. New perspectives on the phase field approach in modeling deformation and material defects as well as microstructural evolution (grain growth, precipitate evolution, solute segregation) are reviewed in [18]. Most of the studies on waves coupled to defect dynamics in elastic media discuss the propagation in isotropic media.

Investigations of waves in anisotropic media are considerably more difficult than the classical and well-understood isotropic problem. The theory of elastic wave behavior propagation in anisotropic solids is well established [19,20]. Extensive theoretical efforts have been made so far to model the effect of heat conduction upon the propagation of plane harmonic waves in anisotropic elastic solids [21–26]. The study of wave propagation in a generalized thermoelastic anisotropic media with additional parameters like prestress, porosity, viscosity, thermal relaxation time, microstructure and other parameters allowed us to obtain vital information about existence of new or modified waves. The eigen-value problems of elastic waves in piezoelectric anisotropic solids were studied by Guo [27]. Valuable attempts have been carried out in [28] to investigate the propagation of waves in a homogeneous, transversely isotropic, piezo-thermoelastic plate. Acharya et al. investigated the general theory of transversely isotropic magneto-elastic interface waves in conducting media under initial hydrostatic tension or compression [29].

The growing applications of new anisotropic materials, especially in the various laser technologies, have encouraged the studies of impact and wave propagation in the anisotopic laser-excited materials and have become very important. In Ref. [30] the DS coupled model has been extended to anisotropic laser-excited solids with atomic defect generation. Propagation of a body plane harmonic wave in an infinite elastic transverse isotropic solid was discussed in particular. It was found that four dispersive wave modes are possible namely, three quasi-elastic wave modes (E) and one quasi-defect concentration wave mode (N). All motions of the medium are found dispersive and coupled with each other due to the defect concentration changes and anisotropic effects.

The present paper is a continuation of our previous work [30]. Its purpose is to study in the context of the DS coupled model the nature of Rayleigh-type surface wave propagation and vibrations in an anisotropic laser-excited solid with a distribution of atomic defects. The author believes that the problem in its present form has not been discussed so far. In the present investigation, frequency equations of coupled elasto-diffusive waves are obtained and some properties of their solutions are discussed. It is found that the surface waves are again found to be dispersive in character. The form of the frequency equations for small values of the diffusion-strain coupled parameter is also deduced and solved analytically. The phase velocity and attenuation coefficients of the waves are influenced by the anisotropic effects. Finally, the derived secular equations in various cases are solved numerically. The computer simulated results for a single crystal of Zn in respect of dispersion curves and attenuation coefficient are presented graphically. The obtained results are in agreement with the corresponding classical results of previous studies for limiting and special cases.

2. BASIC governing equations

Consider an elastic crystalline solid with hexagonal or transverse isotropic symmetry occupying the half-space $z \geq 0$ in Cartesian coordinate system $0xyz$. We assume that the planes of isotropy are perpendicular to z-axis. We choose x-axis in the direction of the propagation of waves so that all particles on a line parallel to y-axis are equally displaced. Thus the motion of the medium is supposed to take place in the xz plane and for the assumed motion the displacement vector $\vec{u}$ has the component $(u,\ 0,\ w)$and all the other variables depend on x, z and t only.

Let us assume that an external energy flux (e.g., laser radiation) generates high concentrations of non-equilibrium atomic defects (vacancies (V-defects) and interstitials (I-defects)) in the near-surface layer (due to heating and renormalization of the defect formation energy). On one hand, the presence of defect density profile results in a force that may induce strain field in medium. On the other hand, when the strain waves propagate, the formation and migration energies of defects change in the compression and dilatation zones; this results in modulations of generation (g) and recombination (r) rates of defects. Therefore, the evolutions of strain and defect concentration fields are inherently coupled.

The dynamical model that can describe the evolution of such a system should be based on: (i) the evolution of atomic defect concentration in a strained solid and (ii) the displacement field of a solid in the presence of a non-uniform defect concentration field. In formulating our theory, we limit our considerations to the case of only one type of atomic defects (for definiteness, I –type defects). Let the medium's temperature be constant (with thermal strains neglected).

Following [30], the constitutive strain–stress-defect relations and 2D field equations in terms of the displacement vector $\vec{u} = (u, 0, w)$ and defect concentration fields $n(x, z, t)$ for a linear transversely isotropic elastic medium, in the absence of the body forces, are

$$\sigma_{xx} = c_{11}u_{,x} + c_{13}w_{,z} - \vartheta_1 n, \tag{1}$$

$$\sigma_{zz} = c_{13}u_{,x} + c_{33}w_{,z} - \vartheta_3 n, \tag{2}$$

$$\sigma_{xz} = 2c_{44}\left(u_{,z} + w_{,x}\right), \tag{3}$$

$$c_{11}u_{,xx} + c_{44}u_{,zz} + (c_{13} + c_{44})w_{xz} - \rho\,\ddot{u} = \vartheta_1 n_{,x}\,, \tag{4}$$

$$(c_{13} + c_{44})u_{xz} + c_{44}w_{,xx} + c_{33}w_{,zz} - \rho\ddot{w} = \vartheta_3 n_{,z}, \tag{5}$$

$$\tau^{-1}n - D_1 n_{,xx} - D_3 n_{,zz} + \dot{n} = g_0\beta\left[\vartheta_1 u_{,x} + \vartheta_3 w_{,z}\right], \tag{6}$$

where c_{ij} are the components of the elasticity tensor, ρ is the density of the medium, D_1, D_3 and ϑ_1, ϑ_3, are respectively, the diffusivities and deformational potentials along and perpendicular to the plane of symmetry; $\tau^{-1} = r$ is the defect recombination rate (τ, relaxation time); g_0 is the defect generation rate constant; $\beta = \left(k_B T\right)^{-1}$. The comma notation is used for spatial derivatives and a superposed dot denotes time differentiation. The terms on the right-hand side of Eq. (6) account for the strain-induced generation of defects. The origin and the physical meaning of these terms are explained in Ref. [1].