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Notes on constraints for the observation of Polar Kerr Effect in complex materials

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ABSTRACT

While Kerr effect has been used extensively for the study of magnetic materials, it is only recently that its has shown to be a powerful tool for the study of more complex quantum matter. Since such materials tend to exhibit a wealth of new phases and broken symmetries, it is important to understand the general constraints on the possibility of observing a finite Kerr effect. In this paper we reviewed the consequences of reciprocity on the scattering of electromagnetic waves. In particular we concentrate on the possible detection of Kerr effect from chiral media with and without time-reversal symmetry breaking. We show that a finite Kerr effect is possible only if reciprocity is broken. Introducing the utilization of the Sagnac interferometer as a detector for breakdown of reciprocity via the detection of a finite Kerr effect, we argue that in the linear regime, a finite detection is possible only if reciprocity is broken. We then discuss possible Kerr effect detection for materials with natural optical activity, magnetism, and chiral superconductivity.

1. Introduction

The discovery by Bednorz and Müller [1] of high temperature (high-Tc) superconductivity in copper oxides (cuprates) had an enormous impact on almost all aspects of research in quantum materials in general, and superconductivity in particular. Soon after this discovery, a whole range of novel superconducting, magnetic and metallic states were discovered in oxides and related systems. New pairing mechanisms associated with novel broken symmetries have been the highlight of the field of superconductivity, and the concept of unconventional superconductivity has emerged. Subsequent discoveries of other novel materials such as carbon nanotubes, graphene, and topological insulators, solidified the importance of quantum matter as a new paradigm in materials physics. Recent progress in the field both, theoretically and experimentally, suggests that local phenomena at the nanometer scale are the key to the novel behavior. The electrons have a very strong propensity to microscopically phase separate and to self-organize in patterns of lower-dimension. These observations also led to the emergence of new experimental techniques that are uniquely capable of probing these new aspects of matter.

In this paper we concentrate on the Kerr effect as a probe for quantum states that exhibit violation of reciprocity. Our approach is to highlight the concept of reciprocity and discussing its role in nonlocal electrodynamics even in the presence of dissipation. Pertaining to the possible observation of Kerr effect, we constraint it to violation of time reversal symmetry because of either spontaneous symmetry breaking such as magnetism or chiral superconductivity, or the application of an external magnetic field. As a consequence, our derivations demonstrate that a Kerr response from a reciprocal material-system such as gyrotropy due to natural optical activity, vanishes exactly.

The organization of this paper is as follows. First we define the Polar Kerr Effect (PKE), which is the primary subject of this paper. We then continue to discuss the concepts of reciprocity and time reversal symmetry (TRS) which are at the heart of understanding the possible observation of a finite PKE. We then apply these concepts to several important examples which include natural optical activity, magnetism and unconventional superconductivity.

2. Polar Kerr Effect

It is customary to define the Kerr effect through the analysis of the state of polarization of light reflected from a magnetic solid,

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hence its MOKE (Magneto-optical Kerr effect) acronym. While the effect was discovered in 1877 by Rev. John Kerr [2], its complete explanation had to wait for the introduction of quantum mechanics. Specifically, spin–orbit coupling and exchange splitting were shown to be necessary for a full understanding of the effect [3]. Of particular interest to us in this paper is the Polar Kerr Effect (PKE) which measures the rotation of a linearly polarized light reflected from a magnetized material at normal incidence.

Assume a material that exhibits a ferromagnetic component of magnetization perpendicular to the surface of a sample. A linearly polarized light that is reflected from that surface will rotate exhibit a rotation of the polarization by a Kerr angle θ_K that reflects the fact that the indices of refraction for right $(+)$ and left $(-)$ circularly polarized light, which make up the linear polarization, are different. Using the convention in which the sense of circular polarization is determined with respect to a given axis (usually the z-axis), the Kerr angle will be determined by comparing the phase shifts of the reflected light of the two circular polarizations, R_{++} and R_{--}. Specifically

$$\theta_K = \frac{1}{2}\left\{\arg[R_{++}] - \arg[R_{--}]\right\} \tag{1}$$

The expression for Kerr effect in terms of the transition amplitudes can then be related to the dielectric function and the respective indices of refraction for the material [3].

$$\theta_K = \Im\left[\frac{\tilde{n}_+ - \tilde{n}_-}{\tilde{n}_+\tilde{n}_- - 1}\right]. \tag{2}$$

Here $\tilde{n}_\pm$ are the complex indices of refraction for right and left circularly polarized light. We can define now the average index of refraction of the material as $\tilde{n} = (\tilde{n}_+ + \tilde{n}_-)/2$, and since in general, $|\tilde{n}_+ - \tilde{n}_-|$âª$|\tilde{n}|$, It can be shown that the above expression can be approximated by

$$\theta_K = -\frac{4\pi}{\omega}\Im\left[\frac{\sigma_{xy}}{\tilde{n}(\tilde{n}^2 - 1)}\right]. \tag{3}$$

Obtaining the behavior of the off-diagonal terms of the electrical conductivity therefore can be used to calculate the Kerr effect in different materials [3].

3. Reciprocity

In the following discussion, as well as in Appendix A and B, we adopt the presentation of Tiggelen and Maynard [4] of a 6-component vector for the wavefunction (3 for the electric field and 3 for the magnetic field). We then assume linear wave propagation according to a Schrödinger-like equation:

$$i\hbar\frac{\partial}{\partial t}\psi(\mathbf{r}, t) = \hat{\mathcal{H}}\psi(\mathbf{r}, t). \tag{4}$$

Here $\hat{\mathcal{H}} = \hat{\mathcal{H}}_0 + \hat{V}$ where $\hat{\mathcal{H}}_0$ is the operator that describes free space wave propagation and $\hat{V}$ describes the scatterer's potential.

Reciprocity is defined by applying an anti unitary operator to the physical process (see e.g. Chapters 5 and 9 in Schiff [5] and Deák and Fülöp in [6]). An antiunitary operator $\hat{K}$ applied to a linear combination of two states, α and β yields

$$\hat{K}(a\psi_\alpha + b\psi_\beta) = a{*}\hat{K}\psi_\alpha + b{*}\hat{K}\psi_\beta \tag{5}$$

which is called antilinearity property. Here "*" denotes complex conjugation. Also, the inverse of $\hat{K}$ coincides with its adjoint $\hat{K}^{-1} = \hat{K}^\dagger$, and the scalar product satisfies

$$\left(\hat{K}\psi_\alpha, \hat{K}\psi_\beta\right) = \left(\psi_\alpha, \psi_\beta\right) \tag{6}$$

Finally, it can be shown that any antiunitary $\hat{K}$ can be written in the form:

$$\hat{K} = \hat{U}\hat{C} \tag{7}$$

where $\hat{U}$ is a unitary operator and $\hat{C}$ is a conjugation operator. Applying $\hat{C}$ to the Schröedinger equation yields the time reversed solution to that equation.

Let us assume that the free hamiltonian commutes with the antiunitary operator $\hat{K}$ such that

$$\hat{K}\hat{\mathcal{H}}_0\hat{K}^{-1} = \hat{\mathcal{H}}_0 \tag{8}$$

But, due to absorption it does not commute with the full Hamiltonian, but rather

$$\hat{K}\hat{V}\hat{K}^{-1} = \hat{V}^\dagger \tag{9}$$

such that

$$\hat{K}\hat{\mathcal{H}}\hat{K}^{-1} = \hat{\mathcal{H}}^\dagger \tag{10}$$

Eqs. (9) and (10) are called the reciprocity condition, and $\hat{K}$ is the reciprocity operator for the system. The above definition becomes clear when $\hat{K}$ is applied to the Green's functions, yielding the familiar reciprocity theorem for the Greens operator.

$$\hat{K}G_E^\pm\hat{K}^{-1} = G_E^{\pm\dagger} \tag{11}$$

Let us first consider the case of elastic scattering such that $\phi_{E_\alpha} \to \phi_{E_\beta}$, but $E_\alpha = E_\beta = E$. Applying $\hat{K}$ to the scattered wavefunction yields

$$\hat{K}\psi_{E_\alpha}^\pm = \psi_{E_\alpha}^{T\mp} \tag{12}$$

which indeed confirms the fact that $\hat{K}$ reverses the scattering process to the reversed one. Thus, the transition amplitudes calculated above will satisfy

$$\langle\beta|\hat{T}|\alpha\rangle = \left(\phi_{E_\beta}, \hat{V}\psi_{E_\alpha}^+\right) = \left([\hat{K}\psi_{E_\alpha}^{T-}], \hat{V}[\hat{K}\phi_{E_\beta}]\right) \equiv \langle\bar{\beta}|\hat{T}|\bar{\alpha}\rangle \tag{13}$$

This is the reciprocity theorem for the transition amplitude as it relates the scattering process to its reversed one.

4. Time reversal symmetry

In the Schröedinger formulation of the wave equation without spin-consideration (appropriate for our discussion of radiation), the time-reversal operation $\hat{\mathcal{T}}$ may be defined as the complex conjugation operator $\hat{C}$, in the position representation [7]. The condition for time reversal symmetry without absorption occurs when

$$\hat{\mathcal{T}}\hat{\mathcal{H}}\hat{\mathcal{T}}^{-1} = \hat{\mathcal{H}} \tag{14}$$

An important example that will be used later is that of a circularly polarized plane wave with wavevector $\mathbf{k}$ and with circular polarization $\sigma = \pm$ for right and left circularly polarized light respectively. Applying $\hat{C}$ we have

$$\hat{C}|\mathbf{k}, \sigma\rangle = |-\mathbf{k}, \sigma\rangle \tag{15}$$