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Plane wave propagation in transversely isotropic laser-excited solids

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ABSTRACT

This paper concentrates on the study of body harmonic wave propagation in an anisotropic laser-excited solid in the context of the model based on coupled equations for the displacement and atomic defect concentration fields. The complex secular equations for transversely isotropic solids are derived and discussed. It is found, in general, four types of dispersive waves, namely a quasi-longitudinal (QL), two quasi-transverse (QT) and a quasi-defect wave (N-mode) can propagate in these types of crystals. The different characteristics of waves like phase velocity, attenuation coefficient are obtained and presented graphically. It is demonstrated that there is an appreciable variation in case of the QL mode as compared with QT and N-modes. Some particular cases have also been discussed.

1. Introduction

Dynamic properties and mechanical behaviors of elastic solids are significant in the ultrasonic inspection of materials, vibrations of structures, in micro- and nanotechnologies and various other fields. Such materials are usually described by equations of linear elasticity. However, there are materials of a more complex microstructure: laser-excited solids with atomic defect (vacancy, interstitial atom) generation, integrated-circuits (IC) interconnect lines with atomic vacancies, granular materials, materials with voids, etc. The linear elasticity theory concept is inadequate for describing the mechanical behavior of these materials. It is necessary to consider the equation describing the changes that occur at the microscopic level in the condensed medium.

The elastic wave propagating in solids carries information about distortions of their form and energy and about the energy losses related to the defect structure; this information is needed for optical–acoustical diagnostics of various parameters and the structure of materials. Also the investigation of generation and diffusion effects in defect's subsystem on elastic wave propagation plays an important role in understanding many laser-induced processes in the solids, particularly in the laser fast recrystallization, laser annealing, multipulse laser etching, and pulsed laser-assisted thin-film deposition.

In works [1–3], the classical theory of elasticity has been extended to isotopic laser-excited solids with atomic defect generation to describe the excitation of instabilities and self-organization of various ordered strain-diffusive structures (micro- and nanosized) on the surfaces and the volume. This theory defect density includes an additional independent kinematic variable and reduces to classical elasticity when the defect diffusion equation and the term related to defect-induced forces in the momentum balance equation are dropped off. The formation of 1D and 2D nonlinear localized coupled strain–defect structures due to concentration–elastic instability were also considered [4–7]. Mathematical models of the above-mentioned studies were based on coupled nonlinear equations for the self-consistent fields of the displacement vector of the medium and atomic defect concentration. However, in this theories temperature is only a parameter. The theory of mechanical waves coupled to atomic defect dynamics and including thermal change effects in pulsed laser-excited solids has been considered by Mirzade [8] and Bargmann and Favata [9]. Some features of the physical problems coupling diffusion, mechanics and thermal waves in a geometrically nonlinear isotropic solid has been studied in Refs. [10–12].

Several researchers have investigated the evolution of stress field due to electromigration in isotropic thin metal films based on 1D vacancy diffusion-mechanical coupled model [13,14]. In this model the electromigration process is assumed to be controlled by a vacancy diffusion mechanism, in which the diffusion takes place by vacancies switching lattice sites with adjacent atoms. A 3D self-consistent model of stress evolution during electromigration has been proposed by Sarychev [15]. In this paper, local volume change

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is assumed to be generated by vacancy diffusion and generation due to electromigration process. The local volume change is then treated as an analog of thermal strain. The stress fields are calculated as a result of volumetric strain induced by electromigration.

In Ref. [16] a plane strain formulation has been derived based on Sarychev's electromigration-deformation constitutive model [15]. The total strain tensor includes the strain due to mechanical loading, the strain due to thermal load, and the volumetric strain due to electromigration. Finite element method is used to simulate the stress distribution in IC interconnect metal lines during electromigration. The obtained simulation results are compared against Blech's [17] experimental results for aluminium lines and an analytical model by Korhonen [13].

Keeping in view the increased usage of anisotropic materials in the development of advanced engineering materials, in the laser additive micro- and nanotechnologies and other fields, the aim of the present investigation is to study the bulk mechanical wave propagation behavior in an infinitely extended anisotropic laser-excited solid with defect generation in the context of coupled concentration–elasticity theory, developed in Refs [1,2]. According to obtained secular equation four types of dispersive waves, namely, a quasi-defect concentration wave (N-mode), a quasi-longitudinal (QL) wave, and two quasi-transverse (QT) waves, can, in general, propagate in considered transversely isotropic media. It is demonstrated that for plane waves propagating in one of the planes of transversely isotropic elastic solid having defect concentration field, only one purely quasi-transverse (QT) wave decouples from the rest of the motion and is not affected by the defect-concentration changes. The other waves are coupled and get modified due to presence of defects. When plane waves are propagating along the axis of the solid, then only longitudinal and defect-concentration waves are coupled, whereas the two quasi-transverse (QT) wave modes get decoupled from the rest of the motion. The general characteristics equation has been solved by using series (perturbation) expansion methods in order to obtain phase velocity and attenuation coefficient of the waves.

2. Statement of the problem and secular equation

We consider an unbounded, anisotropic elastic solid with mobile non-equilibrium atomic point defects (vacancies and interstitial atoms) generated by external energy fluxes (e.g., pulsed laser radiation). We use a fixed Cartesian coordinate system x_i, $i = 1, 2, 3$. Let $\vec{u}(x_1, x_2, x_3, t) = (u_1, u_2, u_3)$ and $N(x_1, x_2, x_3, t)$ be the components of displacement vector and the defect concentration, respectively, of the medium at time t. There could be two types of defect but we limit our consideration to one. We investigate the propagation behavior of bulk mechanical waves, including their variation in phase velocity and dispersion properties.

The basic governing equations for the displacement and defect concentration fields for the case of an anisotropic, linear elastic medium in the absence of body forces, are given by [6,18]:

$$\rho\ddot{u}_i = c_{ijkl}u_{k,lj} - \vartheta_{ik}N_{,k}, \tag{1}$$

$$\dot{N} = -Q_{i,i} + g - rN, \tag{2}$$

$$Q_i = -D_{ik}N_{,k} + Nv_i. \tag{3}$$

Here c_{ijkl} are elastic parameters; ρ is the density of the medium; the deformational tensor ϑ_{ik} controls the strain–defect interaction; Q_i are the components of the defect flux vector; and D_{ik} is the diffusion coefficient of the defects. The functions g and r describe, respectively, the generation and recombination of defects, depending on strain field of the medium.

$$g = g_0 \exp\left(-\beta\vartheta^g_{ik}e_{ik}\right), \quad r = r_0 \exp\left(-\beta\vartheta^m_{ik}e_{ik}\right), \tag{4}$$

where $e_{ij} = (u_{i,j} + u_{j,i})/2$ are the strain tensor components; g_0 is the thermal-fluctuation generation of atomic defects in the absence of the strain field (ϑ^g_{ik} is the deformation potential characterizing the variation of the formation activation energy of defects under the lattice deformation, $\beta^{-1} = k_BT$ (T is the absolute temperature, k_B is the Boltzmann constant)); $r_0 = \tau^{-1}$ is the defect recombination rate in the absence of the strain field (τ is the defect relaxation time, ϑ^m_{ik} is the deformation potential characterizing the variation of the migration activation energy of defects under the lattice deformation). We assume, that g_0, T and r_0 are constants.

The components of the defect drift velocity v_i induced by strain–defect interaction are

$$v_i = D_{ik}\beta\, F_k = -D_{ik}\beta U^{(\text{int})}_{,k} = \beta D_{ik}\vartheta_{jl}e_{jl,k}.$$

We assume that the above constitutive coefficients and the diffusivity tensor satisfy the symmetry relations: $c_{iklm} = c_{lmik}$, $\vartheta_{ik} = \vartheta_{ki}$, and $D_{ik} = D_{ki}$.

In Eqs. (1)–(3) the comma notation is used for partial derivatives and superposed dot is used for time differentiation.

We can express the defect concentration field as $N = N_0 + N^*$ ($N_0 = g_0\tau$ is a spatially homogeneous solution; N^* is a small nonhomogeneous perturbations). Inserting in Eq. (2) and neglecting the nonlinear terms, we get the linearized equation as

$$\frac{\partial N^*}{\partial t} + \tau^{-1}N^* - D_{ij}N^*_{,ij} = g_0\beta\vartheta^{(gm)}_{ik}e_{ik} - \beta N_0 D_{ik}(\vartheta_{jl}e_{jl,k})_{,i}, \tag{5}$$

where $\vartheta^{(gm)}_{ik} = \vartheta^{(g)}_{ik} - \vartheta^{(m)}_{ik}$.

Applying transformation,

$$x_1' = x_1\cos\phi + x_2\sin\phi, \cdots x_2' = -x_1\sin\phi + x_2\cos\phi, \cdots x_3' = x_3, \cdots$$

where ϕ is the angle of rotation in the $x_1 - x_2$ plane, in Eqs. (1) and (4), the basic governing equations for the coupled displacement and defect-concentration fields in transversely isotropic solids can be written as

$$\rho\,\ddot{u}_1 = c_{11}u_{1,11} + c_{12}u_{2,21} + c_{13}u_{3,31} + c_{66}\left(u_{1,22} + u_{2,12}\right) + c_{44}\left(u_{1,33} + u_{3,13}\right) - \vartheta_1 N_{,1}, \tag{6}$$

$$\rho\,\ddot{u}_2 = c_{66}\left(u_{1,21} + u_{2,11}\right) + c_{11}u_{2,22} + c_{44}u_{2,33} + c_{13}u_{3,31} + (c_{13} + c_{44})u_{3,32} - \vartheta_1 N_{,2}, \tag{7}$$

$$\rho\,\ddot{u}_3 = (c_{13} + c_{44})\left(u_{1,13} + u_{2,23}\right) + c_{44}\left(u_{3,11} + u_{3,22}\right) + c_{33}u_{3,33} - \vartheta_3 N_{,3}, \tag{8}$$

$$\dot{N}^* + rN^* - D_1\left(N^*_{,11} + N^*_{,22}\right) - D_3N^*_{,33} = g_0\beta\left[\vartheta_1(u_{1,1} + u_{2,2}) + \vartheta_3u_{3,3}\right], \tag{9}$$

where $D_j = \delta_{ij}D_i$ and $\vartheta_j = \vartheta_i\delta_{ij}$. In the right part of Eq. (9) we have ignored (for simplicity) the influence of strain-induced drift of defects on the behavior of the concentration fields.

For plane harmonic waves, we assume the solution of the form

$$\left(u_k, N^*\right) = \left(\bar{u}_k, \bar{N}\right)\exp\left[i(\xi x_p n_p - \omega t)\right], \tag{10}$$

where ω is the circular frequency (assumed to be real) and ξ is the wave number (in general complex); $\left(\bar{u}_k, \bar{N}\right)$ are the undetermined amplitude vectors that are independent of time (t) and coordinates (x_p). n_p ($p = 1, 2, 3$) the components of the unit wave normal ($\vec{n}$) specifying the direction of wave propagation.