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Dispersion characteristics of electromagnetic excitations in a disordered one-dimensional lattice of coupled microresonators

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ABSTRACT

Localized photonic modes are studied in a non-ideal chain of coupled microcavities with the use of the virtual crystal approximation. The approach proves sufficient to elucidate the effects of varying composition and nearest-neighbor distances on the spectrum. It permits to obtain the density of states of the studied quasiparticles as well as the dispersion dependence of collective excitation frequencies on defect concentration. Based upon the developed description of ideal photonic structures we proceed to study a non-ideal polaritonic crystal constituted by an array of spatially ordered cavities containing atomic clusters. Frequency, effective mass and group velocity of polaritons are analytically derived as functions of vacancy concentration.

1. Introduction

Development of optical quantum information processing devices has created an ever-increasing demand for structures capable of "slowing down" the light [1], i.e. of the effective reduction of group velocity of polaritons. This effect and similar ones occur in a variety of systems such as coupled optical resonators [2,3], solid multilayer semiconductor structures [4], bulk crystals with strong light-matter coupling (GaN, ZnO, see e.g. Refs. [5,6]) etc. There are a number of theoretical and practical problems connected with fabrication of polaritonic crystals [7,8], which present a particular type of photonic crystals [9] featured by a strong coupling between the medium quantum excitations (excitons) and the optical field. The key role in decreasing the group velocity of light is played by the peculiar properties of the so-called "dark" and "bright" polaritons, which are linear superpositions of the photonic states of external electromagnetic fields and macroscopic (coherent) perturbations of a two-level atomic medium [7].

An example of polaritonic structure can be given by a spatially periodic system formed of trapped weakly coupled two-component atomic ensembles interacting with the optical field in a tunnel-coupled cavity array [10]. A distinctive feature of such a structure is the capability of polariton confinement, which is quite analogous to light confinement in photonic crystals (see e.g. [11,12]) and exciton confinement in solid quasiperiodic structures [13–15].

An interest for optical modes in microcavity systems was originally motivated by the advent of optoelectronic devices [16,17] and has been steadily growing over the recent years. In this context defect-based resonators in photonic crystals deserve special mentioning [11]. Ref. [18] demonstrated the attainment of a tight binding between such resonators and quantum dots. Refs. [7,8] gave theoretical analysis of the formation of solitons coupled to lower-dispersion branch (LDB) polaritons in a chain of microresonators; the authors suggest that their results may be of significant importance for quantum information processing. Recent progress in fabrication of quality semiconductor microresonators with Bragg mirrors allowed to attain a Bose–Einstein condensation of LDB-polaritons in quantum wells embedded in semiconductor microcavity structures (CdTe/CdMgTe or GaAs) and to explore their superfluidity [13–15].

Based upon the previously developed concepts of ideal polaritonic structures [7,19] here we investigate the effects of varying eigenfrequencies and nearest-neighbor distances on the dispersion of exciton-like electromagnetic excitations in a one-dimensional lattice of microcavities (coupled resonators with no atomic subsystems). Next, we carry out a numerical modeling of polariton dispersion in a non-ideal chain of coupled microresonators containing impurity atomic clusters.

2. Exciton-like electromagnetic excitations in a non-ideal lattice of coupled microcavities

Unlike in Refs. [7,18,19] they devoted to coupled resonators

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with dopant atoms let us here pose a somewhat different problem. Namely, we shall examine a one-dimensional array of tunnel-coupled randomly distributed microresonators of different types at the total absence of atomic subsystem. Each resonator is assumed to possess a single optical mode. We also account for the overlap of optical fields, which enables photons to move along the chain.

Hamiltonian H of the considered system (see Ref. [7]) is written as

$$H_{ph} = H_{ph}^{0} + H_{int}, \tag{1}$$

where

$$H_{ph}^{0} = \sum_{n} E_n \Psi_n^{+}\Psi_n, \quad H_{int} = -\sum_{n,m} A_{nm} \Psi_n^{+}\Psi_m \tag{2}$$

Indices n and m numerate the sites (one per each unit cell) of the one-dimensional resonator chain. $E_n \equiv \hbar\omega_n$, where ω_n is the photonic mode frequency at the nth site (resonator). Quantity A_{nm} defines the overlap of optical fields of the nth and mth resonators and therefore characterizes the corresponding excitation transfer. Ψ_n^{+} and Ψ_n are Bosonic creation and annihilation operators of the photonic mode respectively. Hamiltonian (1) is formally identical to the excitonic one [20], hence it is natural to refer to the considered electromagnetic excitations as exciton-like.

We assume that the chain of resonators is mainly comprised by the so-called "normal" resonators with "ordinary" values of E_n, A_{nm} while containing a minor admixture of "defect" resonators whose parameters E_n, A_{nm} differ from the "ordinary" ones. Such a crystal admits at least two types of disorderliness: a compositional one (defined by distribution of different-type resonators over the sites) and a topological one (defined by varying nearest-neighbor distances between the sites). Under these circumstances Hamiltonian (1) is not translation invariant, whereas the quantities ω_n and A_{nm} are configurationally dependent. More specifically E_n depends solely on composition, while A_{nm} is dependent both upon composition and resonator locations.

A general recipe for evaluation of quasiparticle spectra in non-ideal systems with randomly distributed elements consists in finding the poles of the configurationally averaged Hamiltonian resolvent [21]. The latter is translation invariant, which permits to characterize the corresponding elementary excitation spectrum by a wave vector $\mathbf{k}$. The necessary calculation can only be carried out in the frames of a certain approximation specific to the studied system. A widespread method of computation of quasiparticle states in imperfect structures is the virtual crystal approximation (VCA) [21,22], which provides an appropriate tool to clarify the spectrum transformations caused by defect concentration variations. Under the VCA the averaged resolvent is identical to the resolvent of the averaged Hamiltonian $\langle H_{ph}\rangle$. In what follows this approximation is used to evaluate and analyze the spectrum of electromagnetic excitations as well as the relevant optical characteristics of the described imperfect lattice.

Since the VCA consists in replacement of configurationally dependent Hamiltonian parameters with their averaged values, Hamiltonian of the "virtual" crystal in our case can be written as

$$\langle H_{ph}\rangle = \sum_{n} \langle E_n\rangle_C \Psi_n^{+}\Psi_n - \sum_{n,m} \langle A_{nm}\rangle_{C,T} \Psi_n^{+}\Psi_m \tag{3}$$

Angular brackets in (3) denote the procedure of configurational averaging, which (on the right-hand side) is carried out over the composition in the first term and over the composition and nearest-neighbor distances in the second term (corresponding index notations are made). Techniques described in Refs. [19,21,23] permit to obtain the following expressions for the averaged quantities of interest:

$$\langle E_n\rangle_C = \sum_{\nu=1}^{s} E^{\nu} C_C^{\nu}; \; \langle A_{nm}\rangle_{C,T} = \sum_{\nu,\mu=1}^{s} \langle A_{nm}^{\nu\mu}\rangle_T C_C^{\nu} C_C^{\mu}, \tag{4}$$

where $\langle A_{nm}^{\nu\mu}\rangle_T$ defines the electromagnetic excitation transfer from a νth type of resonator at the nth site to a μth type of resonator at the mth site of the "virtual" crystal, C_C^{ν}, C_C^{μ} are concentrations of the νth and μth types of resonators and $\sum_{\nu=1}^{s} C_C^{\nu} = 1$ and s is the number of resonator types. Configurational averaging "restores" the translation invariance of resonator system, which permits to invoke a wave vector $\mathbf{k} = (k, 0, 0)$ for description of eigenvalues and eigenfunctions of Hamiltonian (3).

Diagonalization of Hamiltonian (3) via Bogolyubov–Tyablikov transformation [20] leads to the following expressions for excitation energies and overlap parameters of the optical fields of adjacent resonators:

$$E(k, \{C_C\}, \{C_T\}) = \sum_{\nu=1}^{s} E^{\nu} C_C^{\nu} - \sum_{\nu,\mu=1}^{s} A^{\nu\mu}(k, \{C_T\}) C_C^{\nu} C_C^{\mu}, \tag{5}$$

$$A^{\nu,\mu}(k, \{C_T\}) = \sum_{m} \langle A_{nm}^{\nu\mu}\rangle_T \exp\left[ika_{nm}(\{C_T\})\right] \tag{6}$$

In (6) $a_{nm}(\{C_T\}) = a(\{C_T\})(n-m)$, where $a(\{C_T\})$ is the averaged period of the isoperiodic "virtual" one-dimensional resonator chain, $\{C_C\}$, $\{C_T\}$ are the sets of types and locations of resonators. From Eq. (5) it follows that the dispersion law $\omega(k)$ of electromagnetic excitations in the considered system is determined by the frequency characteristics of resonators as well as by the explicit form of matrix $A^{\nu\mu}(k, \{C_T\})$.

To fix our ideas let us examine electromagnetic excitations in a binary chain consisting of two types of resonators ($s = 2$). We assume that they are arbitrarily distributed and separated by the distances a_1 and a_2. If so, expression (5) takes the form

$$\omega(k, \{C_C\}, \{C_T\}) = \sum_{\nu=1}^{2} \omega^{\nu} C_C^{\nu} - \frac{1}{\hbar} \sum_{\nu,\mu}^{2} A^{\nu\mu}(k, \{C_T\}) C_C^{\nu} C_C^{\mu} \tag{7}$$

Next, the Fourier-transform of the matrix $A^{\nu,\mu}(k, \{C_T\})$ appearing in (6) can in the nearest-neighbor approximation [19] be written as:

$$A^{\nu\mu}(k, \{C_T\}) = 2A^{\nu\mu}[a(\{C_T\})] \cos ka(\{C_T\}) \tag{8}$$

As for the period $a(\{C_T\})$ of the "virtual" 1D resonator chain it is readily found to be $a(\{C_T\}) = C_{1T}a_1 + C_{2T}a_2$, where the obvious condition $C_{1T} + C_{2T} = 1$ must hold. Dependence $A^{\nu\mu}[a(\{C_T\})]$, which determines the transfer probability of electromagnetic excitations between neighboring resonators can in the frames of the developed model be written as $A^{\nu\mu}[a(\{C_T\})] = A^{\nu\mu}(a_1) \exp\left[|a_1 - a(\{C_T\})|/a_1\right]$. Quantities $A^{\nu\mu}(a_1)$ characterize the overlap of optical fields of neighboring resonators in an ideal chain of period a_1. The latter is taken as the reference one for the subsequent variation of distances. Under these circumstances the dispersion law $\omega(k, C_C, C_T)$ takes on the form

$$\begin{aligned} \omega(k, C_C, C_T) &= \omega_1 + (\omega_2 - \omega_1)C_C \\ &\quad - \left[A^{11}(1-C_C)^2 + A^{22}C_C^2 + \left(A^{12} + A^{21}\right)(1-C_C)C_C\right] \\ &\quad \times \frac{2}{\hbar} \exp\left(-\frac{C_T|a_1 - a_2|}{a_1}\right) \cos k\left[a_1 + C_T(a_2 - a_1)\right] \end{aligned} \tag{9}$$

where notations are adjusted as $C_{2C} \equiv C_C$, $C_{2T} \equiv C_T$.

Numerical evaluation of Eq. (9) requires the assignment of modeling frequencies of resonance photonic modes pertaining to the first and the second types of resonators; these we take to be $\omega_1 = 2\pi \times 25.0\ \text{THz} \approx 157 \times 10^{12}\ \text{Hz}$ and