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Optical properties of plasmons in a multiple quantum well semiconductor superlattice under electric and magnetic fields

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ABSTRACT

The behavior of multiple quantum well GaAs/$Al_xGa_{1-x}As$ semiconductor superlattices with different dielectric interfaces are considered under magnetic and electric fields perpendicular and parallel to the superlattice axis, respectively. The parabolic confining potential well was varied with the compositional rate of the $Al_xGa_{1-x}As$ barrier. Taking into account intrasubband and intersubband transitions and using random phase approximation, the density-density correlation function is calculated as a function of the magnetic field strength, compositional rate, and averaged electric field strength over the quantum well. In this way, the dispersion of the surface and bulk state energies are obtained. The Raman intensities for these states are also obtained as a function of incoming light energy for various averaged electric field strengths over the quantum well.

1. Introduction

Studies of semiconductor superlattices fabricated by advanced growth technologies have recently been of great theoretical and experimental interest [1–12]. A GaAs/$Al_xGa_{1-x}As$ superlattice [7,9,10,12] consisting of n-GaAs polar semiconductors separated by $Al_xGa_{1-x}As$ barriers is a way of tuning the properties of a quasi-two-dimensional electron gas within a quantum well. In order to study the electrical and optical properties of such superlattices, it is important to understand the possible collective excitations found in inelastic light scattering experiments [13–17] and the observed dispersion relations of surface and bulk modes. These studies have mainly been restricted to systems under applied magnetic fields. A more systematic and theoretical investigation of the properties of superlattices in the simultaneous presence of magnetic and electric fields is needed.

The purpose of this work was to investigate the electronic collective modes of GaAs/$Al_xGa_{1-x}As$ semiconductor superlattices under magnetic and electric fields, parallel and perpendicular to the superlattice axis, respectively, while also tuning the parabolic confining potential well by changing the compositional rate of the $Al_xGa_{1-x}As$ barrier [18]. Using a model system, we obtained the dispersion relations for bulk and surface plasmons, due to the intrasubband and intersubband transitions, as a function of magnetic field strength, compositional rate, and averaged electric field strength over the quantum well [12]. We also obtained the Raman intensities expected for these bulk and surface plasmons as a function of incoming light energy for the various averaged electric field strengths over the quantum well.

In Sec. II, we present an idealized model for the dependence of the parabolic confining potential well on compositional rate of the $Al_xGa_{1-x}As$ barrier under magnetic and electric fields. Using the random phase approximation for a semi-infinite superlattice, the density-density correlation function is derived in Sec. III, which includes a calculation of intrasubband and intersubband scattering of incoming light as a function of magnetic field strength, compositional rate, and averaged electric field strength over the quantum well. In Sec. IV, the dispersions of bulk and surface plasmons are calculated as a function of magnetic field strength, compositional rate, and averaged electric field strength. In Sec. V, the Raman spectra of bulk and surface plasmons are calculated for various averaged electric field strengths over the quantum well. Conclusions are given in the last section.

2. Model

A simple model for a multiple quantum well (MQW) in the GaAs/$Al_xGa_{1-x}As$ superlattice system consists of a parabolic confining potential well in the z-direction, parallel to the superlattice

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axis, and assumes free electron motion in the x and y directions. When n electrons per unit cell occupy a quantum well, they move in response to an external dc bias voltage. In this calculation, we assume an averaged effective electric field [12] over the entire quantum well. This assumption is valid since the Thomas–Fermi screening length [19,20] of 6.46 nm, calculated for a well width of $L=30$ nm, electronic density $n_e = 7.7\times10^{11}$ cm^{-2}, and effective electron mass $m_0 = 0.0665m_e$ where m_e is the bare electron mass [7,9,10,12,16], is much smaller than the quantum well width. In the presence of an averaged electric and a static magnetic field, the one-electron Hamiltonian in the quantum well is expressed in a unified manner as [12]:

$$H=\frac{1}{2}[\vec{p}+e\vec{A}]\begin{pmatrix}1/m_t & 0 & 0\\ 0 & 1/m_t & 0\\ 0 & 0 & 1/m_l\end{pmatrix}\left[\vec{p}+e\vec{A}\right]+\frac{1}{2}m_l\omega_z z^2+e\vec{F}\times\vec{z}, \tag{1}$$

where $\vec{F}$ is the averaged electric field strength over the quantum well, $m_l\omega_z{}^2z^2/2$ is a parabolic potential well, $\vec{p}$ is the momentum operator, $\vec{A}$ is the vector potential for constant magnetic field $\vec{B}=\vec{\nabla}\times\vec{A}$, and m_t and m_l represent the transverse and the longitudinal mass components, respectively. For a magnetic field $\vec{B}=(B,0,0)$ in the Landau gauge $\vec{A}=(0,-zB,0)$, the Hamiltonian under static magnetic and electric field perpendicular and parallel, respectively, to the superlattice axis can be rewritten as:

$$H=\frac{p_x{}^2}{2m_t}+\frac{p_y{}^2}{2\tilde{m}}+\frac{p_z^2}{2m_l}+\frac{1}{2}m_l\Omega_z{}^2(z-z_0)^2+\frac{2p_y\omega_c eF-e^2F^2}{2m_l\Omega_z{}^2}, \tag{2}$$

where $\omega_c=eB/m_t$ is the cyclotron frequency, $\Omega_z^2=\omega_z^2+m_t\omega_c{}^2/m_l$, $\tilde{m}=m_t\Omega_z{}^2/\omega_z{}^2$, $z_0=(p_y\omega_c-eF)/(m_l\Omega_z{}^2)$, and p_x and p_y are the x- and y-directional momentum operators, respectively. The eigenfunctions of Eq. (2) have the form $\exp(iq_xx)\exp(iq_y'y)\varphi(z-z_0)$ with $q_y'=(\omega_z/\Omega_z)q_y$. Here q_x and q_y are the quasi-continuous wave vectors in the x and y directions, respectively. The $\varphi(z)$ are the eigenfunctions for a simple harmonic oscillator. The energy eigenvalues are:

$$E_n(q_x,q_y)=(n+(1/2))\hbar\Omega_z+\frac{\hbar^2q_x^2}{2m_t}+\frac{\hbar^2q_y{}^2}{2\tilde{m}}+eFz_0+\frac{m_t{}^2\omega_c{}^2V_d{}^2}{2m_l\Omega_z^2}, \tag{3}$$

where $n(=0,1,2)$ denotes the quantum number and $V_d=F/B$. The cyclotron radius z_0 decreases with increasing magnetic field strength B and increasing averaged electric field. The confinement frequency Ω_z in Eq. (3) satisfies the condition $1.247x/2.0=m_l\Omega_z^2(L/2)^2/2$ with the compositional fraction x of the $Al_xGa_{1-x}As$ barriers [18]. The direct energy gap is given by: $E_g=1.424+1.247x$ eV. Using this notation, we assume up to a third subband level with $E_n=(n+(1/2))\hbar\Omega_z$ and a maximum well width L for the parabolic confining potential.

3. Density-density correlation function

We consider a MQW superlattice consisting of N unit cells, in which there is an electron layer in each cell. The particle state in the unit cell can then be assumed to be of the form:

$$|qnl\rangle=\exp[i(q_xx+q_y'y)]\varphi_{nl}(z-z_0-la)\exp(iq_zla), \tag{4}$$

where $\varphi(z-z_0-la)$ is the electron wave function in the lth quantum well, which is chosen to be real, and $l(=0,1,\ldots,N-1)$ is the cell index. The density-density correlation function $\prod(q,\omega,z,z')$ calculated by Hawrylak et al. [11] for a semi-infinite MQW superlattice can be obtained from Eq. (4) as:

$$\prod(q,\omega,z,z')=\sum_{n,n',m,m'}\sum_{l,l_1,l',l_1'}\prod_{nn'mm'}(l,l_1,l',l_1')\Phi^*_{nn'll_1}(z)\Phi_{mm'l'l_1'}(z), \tag{5}$$

where $\Phi_{nn'll_1}(z)=\varphi^*_{nl}(z)\varphi_{n'l_1}(z)\exp[-iq_z(l-l_1)a]$. The quantity $\prod_{nn'mm'}(q_z,q_z')$ obtained by Fourier transform of the quantity $\prod_{nn'mm'}(l,l')$ in Eq. (5) can be expressed in matrix form as (q_z,q_z'), which satisfies an integral equation in the random-phase approximation: $(q_z,q_z')={}^0\delta_{q_z,q_z'}+\sum_{q''_z}{}^0\underline{V}(q_z,q''_z)(q''_z,q_z')$. Here, ^{0} is the layer-independent polarizability of the noninteracting system, expressed as $\Pi^0_{nm}\delta_{nn'}\delta_{mm'}$. $\underline{V}(q_z,q_z')$ is the Coulomb interaction between layers and is decomposed in the usual way through Fourier transform: $V_{nn'rs}(l,l')=\sum_{l_1,l_1'}V_q\int_{-\delta}^{\infty}dz\int_{-\delta}^{\infty}dz'\Phi_{nn'll_1}(z)[\exp(-q|z-z'|)+\alpha_1\exp(-q(z-z'))]\Phi_{l'l_1'rs}(z')$. This expression for $\underline{V}(q_z,q_z')$ also takes into account the effect of image charges and eigenstates in the wells [11] through the parameter δ, defined as the distance from the first layer to the interface. Here, $q=(q_x^2+q_y'2)^{1/2}$, $\alpha_1=[(\varepsilon-\varepsilon_0)/(\varepsilon+\varepsilon_0)]\exp(-2\delta q)$, and $V_q=2\pi e^2/\varepsilon q$. The bulk and surface parts of this expression for the Coulomb potential are:

$$\underline{V}^B(q_z)=\frac{V_q}{2}[(\sinh(qa)/P(q_z)-1)(\underline{gg}_{\mp}(q)+\underline{g}^0)], \tag{6}$$

and

$$\underline{V}^S(q_z,q_z')=\frac{[1-\exp(-qaN)]}{4NP(q_z)P(q_z')}[\underline{a}^{11}+\underline{a}^{12}\exp(-iq_z'a)+\underline{a}^{21}\exp(iq_za)+\underline{a}^{22}\exp(i(q_z-q_z')a)], \tag{7}$$

where q_z and $q_z'=2\pi P/Na(P=0,1,2,\ldots,N-1)$, $P(q_z)=\cosh(qa)-\cos(q_za)$, $\underline{gg}_{\mp}(q)=g_{nn'}(-q)g_{mm'}(q)+g_{mm'}(-q)g_{nn'}(q)$ with $g_{nn'}(\pm q)=\int\varphi_n(z)\varphi_{n'}(z)\exp(\pm qz)dz$, and $\underline{g}^0=\iint[\varphi_n^2(z)\varphi_m^2(z')+\varphi_m^2(z)\varphi_n^2(z')]\exp(-q|z-z'|)dzdz'$. Here $\underline{a}^{11}=\underline{gg}_{\mp}+\alpha_2e^{2qa}\underline{gg}_{-}$, $\underline{a}^{12}=\underline{a}^{21}=-\cosh(qa)\underline{gg}_{\mp}-\alpha_2e^{qa}\underline{gg}_{-}$, and $\underline{a}^{22}=\underline{gg}_{\mp}+\alpha_2\underline{gg}_{-}$ with $\alpha_2=\alpha_1(1-\exp(-qaN))$, and $\underline{gg}_{-}=g_{nn'}(-q)g_{mm'}(-q)$ [11,12]. Using the self-consistent linear density-density theory, a dielectric matrix, $\varepsilon(nm;ll')=\underline{\Pi}^0/\underline{\Pi}(q_z,q_z')$, can be obtained through the elements of the polarizability tensor. Similarly, the matrix form of (q_z,q_z') yields two coupled equations for ${}^B(q_z)$ and ${}^S(q_z,q_z')$, corresponding to bulk and surface parts, respectively, as:

$${}^B(q_z)=[1-{}^0\underline{V}^B(q_z)]^{-1}{}^0, \tag{8}$$

and

$${}^S(q_z,q_z')=\frac{(1-\exp(-qaN)){}^B(q_z)}{4NP(q_z)P(q_z')}[\underline{A}^{11}+\underline{A}^{12}\exp(-iq_z'a)+\underline{A}^{21}\exp(iq_za)+\underline{A}^{22}\exp(i(q_z-q_z')a)]{}^B(q_z'), \tag{9}$$

The elements of $\underline{A}^{ij}$ are related to the elements of $\underline{a}^{ij}$ in Eq. (7) as follows:

$$\begin{pmatrix}\underline{a}^{11} & \underline{a}^{12}\\ \underline{a}^{21} & \underline{a}^{22}\end{pmatrix}=\begin{pmatrix}(1-\underline{a}^{11}\underline{G}-\underline{a}^{12}\underline{H}^-) & -(\underline{a}^{11}\underline{H}^+ +\underline{a}^{12}\underline{G})\\ -(\underline{a}^{21}\underline{G}+\underline{a}^{22}\underline{H}^-) & (1-\underline{a}^{21}\underline{H}^+ -\underline{a}^{22}\underline{G})\end{pmatrix}\begin{pmatrix}\underline{A}^{11} & \underline{A}^{12}\\ \underline{A}^{21} & \underline{A}^{22}\end{pmatrix}=MA. \tag{10}$$

Here, $\underline{G}=(1-\exp(-qaN))/(4N)\sum_{q_z}[{}^B(q_z)/P(q_z)^2]$ and $\underline{H}^{\pm}=(1-\exp(-qaN))/(4N)\sum_{q_z}[\exp(\pm iq_zz){}^B(q_z)/P(q_z)^2]$ [11,12]. Using the trans-formation introduced by Jain and Allen [4] for a semi-infinite array of electron gas layers, $\varepsilon(nm;ll')^{-1}$ for $\varphi_{nl}^2(z)\varphi_{ml'}^2(z)$ is:

$$\varepsilon(nm;ll')^{-1}=\frac{1}{\gamma_{nnmm}}\left\{\delta_{ll'}+\frac{\prod_{nm}^0V_q\sinh(qa)gg_{nm\mp}}{2\gamma_{nnmm}\sqrt{(b_{nm}^2-1)}}U_{nnmm}^{-|l-l'|}\right.$$