Physica B 451 (2014) 20–25

Contents lists available at ScienceDirect

Physica B

journal homepage: www.elsevier.com/locate/physb

Electron gas high-frequency conductivity on the surface of a nanotube with superlattice in magnetic field

A.M. Ermolaev, G.I. Rashba *

Department of Theoretical Physics, V.N. Karazin Kharkiv National University, 4 Svobody square, 61022 Kharkiv, Ukraine

ARTICLE INFO

Article history:
Received 26 September 2013
Received in revised form
10 June 2014
Accepted 12 June 2014
Available online 21 June 2014

Keywords:
Nanotube
Superlattice
Magnetic field
Conductivity tensor
Aharonov–Bohm and de Haas–van Alphen oscillations
Beatings

ABSTRACT

Kubo formula was obtained for conductivity tensor of electron gas on the surface of nanotube with superlattice in magnetic field. The high-frequency conductivity tensor components were calculated for quantum and quasiclassical cases. Electromagnetic wave Landau damping areas in the tube were determined. The conductivity tensor components show Aharonov–Bohm type oscillations and de Haas–van Alphen ones. When Fermi energy exceeds the miniband width, beatings are observed in the plot of conductivity vs. the tube parameters. Otherwise, the beatings are absent.

1. Introduction

Interest in carbon [1–3] and semiconductor [4–6] nanotubes is generated by their unique properties, namely high strength and conductivity, as well as magnetic, waveguide, and optical properties. These systems are prepared by rolling up a graphene sheet (or two-dimensional heterostructure) into a tube. Depending on the rolling up manner, the tube has metallic, semiconductor, or dielectric properties.

Modern technologies allow creating not only nanotubes, but nanotubes with superlattices. Along with flat superlattices [7–15], also ones with cylindrical symmetry exist [16]. They are of radial and longitudinal types [16,17]. The radial superlattice is a set of coaxial cylinders, while the longitudinal one looks like a set of coaxial rings of the same radius. The tubes with longitudinal superlattice are prepared by lithographic methods, using embedding the fullerenes into the tube. In such system, there exists the periodic potential acting upon electrons moving along the tube. In the electron energy spectrum the minibands appear. The electron density of states has root singularities at the miniband boundaries [18].

In connection with increased interest in currents within the cylindrical conductors, the authors of Ref. [19] have calculated the longitudinal conductivity for solid and hollow cylinders without superlattice in magnetic field, and considered quantum electromagnetic waves in such systems. Exact expressions for all the components of the conductivity tensor for degenerate and nondegenerate electron gas on the nanotube surface without superlattice are presented in Ref. [20]. It is worth to be clarified, how the superlattice affects this tensor. In the present paper, the dynamic conductivity tensor components were calculated based on the model of effective mass for the nanotube with longitudinal superlattice in magnetic field. The superlattice axis and the magnetic field vector were considered to be parallel to the tube axis. In Section 2, Kubo formula was obtained for conductivity tensor. In Sections 3 and 4, the conductivity tensor components were calculated for quantum and quasiclassical cases. In Section 5, the results are summarized.

2. Conductivity tensor

For the nanotube with superlattice in magnetic field, the surface electron gas linear response to an electromagnetic wave

$$\vec{E} = \vec{E}_0 \exp i(m\varphi + qz - \omega t)$$

is characterized by conductivity two-dimensional tensor $\sigma_{\alpha\beta}(m, q, \omega)$. Here $\vec{E}$ is electric field of the wave, m is integer number, q and ω are the wave vector and frequency of the wave, φ and z are cylindrical coordinates. The density of surface current on the

* Corresponding author.
E-mail addresses: alexander.m.ermolaev@univer.kharkov.ua (A.M. Ermolaev), georgiy.i.rashba@univer.kharkov.ua (G.I. Rashba).

http://dx.doi.org/10.1016/j.physb.2014.06.016
0921-4526/

tube is

$$j_\alpha(m,q,\omega)=\sum_\beta \sigma_{\alpha\beta}(m,q,\omega)E_\beta(m,q,\omega), \quad (1)$$

where $j_\alpha(m,q,\omega)$ and $E_\beta(m,q,\omega)$ are cylindrical harmonics of $\vec{j}$ and $\vec{E}$ vectors. Kubo's formula for the conductivity tensor of electron gas on the surface of the nanotube with superlattice is [20]

$$\sigma_{\alpha\beta}(m,q,\omega)=i\frac{e^2n}{m_*\omega}\delta_{\alpha\beta}+\frac{1}{\omega}\int_0^\infty dt\, e^{i\omega t}\langle[J_\alpha(m,q,t),J_\beta(-m,-q,0)]\rangle, \quad (2)$$

where m_* and e are electron effective mass and charge respectively , n is surface density of electrons, and $\vec{J}(m,q,t)$ is the cylindrical harmonics of current density operator in the external magnetic field $\vec{B}$. The angle brackets denote the average value of the operator commutator. The quantum constant was assumed as unity. The components of $\vec{J}(m,q)$ vector are

$$J_\varphi=-\frac{2e}{m_*a\sqrt{S}}\sum_{lk}\left(l+\eta+\frac{m}{2}\right)a^+_{lk}a_{(l+m)(k+q)},$$

$$J_z=-\frac{2e}{m_*\sqrt{S}}\sum_{lk}\left(k+\frac{q}{2}\right)a^+_{lk}a_{(l+m)(k+q)}, \quad (3)$$

where l and k are projections of electron angular momentum and momentum, respectively, onto the axis of the tube with radius$a$, a_{lk} and a^+_{lk} are operators of annihilation and creation of electrons in $lk\rangle$ state, $\eta=\Phi/\Phi_0$ is the ratio of magnetic flux $\Phi=\pi a^2B$ through the tube cross-section to the flux quantum $\Phi_0=2\pi c/e$ [21] and $S=2\pi aL$ is the surface area for the tube with length$L$. Spin splitting of levels is not considered in Eq. (3).

From Expressions (2) and (3) with taking into account Wick theorem we obtain the components of conductivity tensor:

$$\sigma_{\varphi\varphi}=i\frac{e^2n}{m_*\omega}+i\frac{2e^2}{m_*^2a^2\omega S}\sum_{lk}f(\varepsilon_{lk})\left[\frac{(l+\eta+\frac{m}{2})^2}{\varepsilon_{lk}-\varepsilon_{(l+m)(k+q)}+\omega+i0}-\frac{(l+\eta-\frac{m}{2})^2}{\varepsilon_{(l-m)(k-q)}-\varepsilon_{lk}+\omega+i0}\right], \quad (4)$$

$$\sigma_{\varphi z}=\sigma_{z\varphi}=i\frac{2e^2}{m_*^2a\omega S}\sum_{lk}f(\varepsilon_{lk})\left[\frac{(l+\eta+\frac{m}{2})(k+\frac{q}{2})}{\varepsilon_{lk}-\varepsilon_{(l+m)(k+q)}+\omega+i0}-\frac{(l+\eta-\frac{m}{2})(k-\frac{q}{2})}{\varepsilon_{(l-m)(k-q)}-\varepsilon_{lk}+\omega+i0}\right], \quad (5)$$

$$\sigma_{zz}=i\frac{e^2n}{m_*\omega}+i\frac{2e^2}{m_*^2\omega S}\sum_{lk}f(\varepsilon_{lk})\left[\frac{(k+\frac{q}{2})^2}{\varepsilon_{lk}-\varepsilon_{(l+m)(k+q)}+\omega+i0}-\frac{(k-\frac{q}{2})^2}{\varepsilon_{(l-m)(k-q)}-\varepsilon_{lk}+\omega+i0}\right] \quad (6)$$

Here f is Fermi function, ε_{lk} is electron energy on the tube surface. That is

$$\varepsilon_{lk}=\varepsilon_0(l+\eta)^2+\Delta(1-\cos kd), \quad (7)$$

where $\varepsilon_0=(2m_*a^2)^{-1}$ is rotational quantum, Δ and d are amplitude and period of modulating potential on the tube surface, respectively. The first term in Eq. (7) was obtained in [21]. The second addend in the right part of (7) is taken from the theory of tight binding of electrons with the lattice [22]. This is often used in the theory of semiconductor superlattices [23–26]. The real parts of the components $\sigma_{\varphi\varphi}$ and $\sigma_{zz}$are even functions of m and ω, while imaginary parts are odd ones.

At zero temperature in summation $\sum$the values k in Formulas (4)–(6) are limited to interval $-k_l\le k\le^k k_l$, where

$$k_l=\frac{1}{d}\arccos\frac{\varepsilon_l+\Delta-\mu}{\Delta}$$

is the maximum momentum of the electrons in the miniband l, $\varepsilon_l=\varepsilon_0(l+\eta)^2$ is the miniband boundaries, μ is the Fermi energy. If $q=0$, at zero temperature from Formulas (4)–(6) we calculate the components of dynamical conductivity tensor:

$$\mathrm{Re}\sigma_{\varphi\varphi}(m,\omega)=\frac{e^2}{\pi m_*^2a^3\omega}\sum_l k_l\left[\left(l+\eta+\frac{m}{2}\right)^2\delta(\omega-\Omega_+)-\left(l+\eta-\frac{m}{2}\right)^2\delta(\omega-\Omega_-)\right],$$

$$\mathrm{Im}\sigma_{\varphi\varphi}=\frac{e^2n}{m_*\omega}+\frac{e^2}{\pi^2m_*^2a^3\omega}\sum_l k_l\left[\frac{(l+\eta+\frac{m}{2})^2}{\omega-\Omega_+}-\frac{(l+\eta-\frac{m}{2})^2}{\omega-\Omega_-}\right], \quad (8)$$

$$\sigma_{\varphi z}(m,\omega)=0,$$

$$\mathrm{Re}\sigma_{zz}(m,\omega)=\frac{e^2}{3\pi m_*^2a\omega}\sum_l k_l^3[\delta(\omega-\Omega_+)-\delta(\omega-\Omega_-)],$$

$$\mathrm{Im}\sigma_{zz}(m,\omega)=\frac{e^2n}{m_*\omega}+\frac{e^2}{3\pi^2m_*^2a\omega}\sum_l k_l^3\left[\frac{1}{\omega-\Omega_+}-\frac{1}{\omega-\Omega_-}\right]. \quad (9)$$

Here

$$\Omega_\pm=\varepsilon_0m[2(l+\eta)\pm m]$$

are frequencies of direct transitions of electrons between the miniband boundaries ε_l in the field of electromagnetic wave. During the transitions, conservation laws for longitudinal components of angular momentum, momentum and energy are satisfied.

3. Degenerated electron gas in the quantum limit

At zero temperature, the summation over l in Eqs. (8) and (9) is limited by the condition $|\varepsilon_l+\Delta-\mu|\le\Delta$. This means that Fermi energy is concentrated within the miniband. The minibands are positioned in the intervals $[\varepsilon_l,\varepsilon_l+2\Delta]$ and have the width2Δ.

Generally, the semiconductor nanotubes with radius $a=(10^{-7}-10^{-6})$ cm in magnetic field $B=10^5$ G are used. In this case, the electrons of the semiconductor nanotube occupy little quantity of bottom minibands, which boundaries at $\eta<1/2$satisfy the inequality$\varepsilon_0\eta^2<\varepsilon_{-1}<\varepsilon_1<\varepsilon_{-2}<\dots$ In the quantum limit where $n<1/\pi ad$, Fermi energy is concentrated in the bottom miniband $l=0$ $[\varepsilon_0\eta^2,\varepsilon_0\eta^2+2\Delta]$. In this case, in the absence of spatial dispersion, from Eqs. (8) and (9) we obtain

$$\mathrm{Re}\sigma_{\varphi\varphi}=\frac{e^2k_0}{\pi m_*^2a^3\omega}\left[\left(\eta+\frac{m}{2}\right)^2\delta(\omega-\varepsilon_0m(2\eta+m))-\left(\eta-\frac{m}{2}\right)^2\delta(\omega-\varepsilon_0m(2\eta-m))\right],$$

$$\mathrm{Im}\sigma_{\varphi\varphi}=\frac{e^2n}{m_*\omega}+\frac{e^2k_0}{\pi^2m_*^2a^3\omega}\left[\frac{(\eta+\frac{m}{2})^2}{\omega-\varepsilon_0m(2\eta+m)}-\frac{(\eta-\frac{m}{2})^2}{\omega-\varepsilon_0m(2\eta-m)}\right], \quad (10)$$

$$\mathrm{Re}\sigma_{zz}=\frac{e^2k_0^3}{3\pi m_*^2a\omega}[\delta(\omega-\varepsilon_0m(2\eta+m))-\delta(\omega-\varepsilon_0m(2\eta-m))],$$

$$\mathrm{Im}\sigma_{zz}=\frac{e^2n}{m_*\omega}+\frac{e^2k_0^3}{3\pi^2m_*^2a\omega}\left[\frac{1}{\omega-\varepsilon_0m(2\eta+m)}-\frac{1}{\omega-\varepsilon_0m(2\eta-m)}\right]. \quad (11)$$

Here $\Omega_\pm=\varepsilon_0m(2\eta\pm m)$. The superlattice parameters Δ and d are included in Eqs. (10) and (11) only via the maximum momentum k_0 of electrons in the bottom miniband. In the absence of superlattice, $\Delta\to\infty$, $d\to 0$, $d^2\Delta\to m_*^{-1}$. Then

$$k_l=[2m_*(\mu-\varepsilon_l)]^{1/2},$$

and Eqs. (10) and (11) agree with ones obtained in Ref. [20]. At $m=0$, only the imaginary part $e^2n/m_*\omega$ remains in Eqs. (10) and (11), while the real part is zero. This determines the electromagnetic wave energy absorbed by electrons. In the absence of direct and indirect transitions of electrons, the absorption is zero.

As the electron density grows, the number of addends in Eqs. (8) and (9) increases. If Fermi energy is concentrated in the second miniband, the oscillator forces of electron resonance transitions in Eqs. (8) and (9) are determined by values k_0 and k_{-1}. These are included in Eqs. (8) and (9), if the minibands are overlapped, i.e.