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Band structures in two-dimensional phononic crystals with periodic Jerusalem cross slot

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ABSTRACT

In this paper, a novel two-dimensional phononic crystal composed of periodic Jerusalem cross slot in air matrix with a square lattice is presented. The dispersion relations and the transmission coefficient spectra are calculated by using the finite element method based on the Bloch theorem. The formation mechanisms of the band gaps are analyzed based on the acoustic mode analysis. Numerical results show that the proposed phononic crystal structure can yield large band gaps in the low-frequency range. The formation mechanism of opening the acoustic band gaps is mainly attributed to the resonance modes of the cavities inside the Jerusalem cross slot structure. Furthermore, the effects of the geometrical parameters on the band gaps are further explored numerically. Results show that the band gaps can be modulated in an extremely large frequency range by the geometry parameters such as the slot length and width. These properties of acoustic waves in the proposed phononic crystals can potentially be applied to optimize band gaps and generate low-frequency filters and waveguides.

1. Introduction

Over the last two decades, the propagation of acoustic and elastic waves in periodic composite materials, known as phononic crystals (PCs) has attracted considerable attention for their abundant physics and potential engineering applications [1–4]. Phononic crystals are periodic artificial composite materials made of two or more materials with different elastic constants, and they can demonstrate various novel physical properties; in particular, the existence of phononic band gaps (BGs), in which the propagation of elastic waves is prohibited [5–8]. With the existence of BGs, phononic crystals possess extensive potential applications, such as vibration and noise reduction [9–11], sound filters [12,13] and waveguides [14,15].

Earlier studies have demonstrated that the occurrence of the BGs is attributed to Bragg scattering and localized resonances. For the first mechanism, the BGs are attributed to the destructive interference between incident acoustic waves and reflections from the periodic scatterers. When wavelengths are of the order of the lattice constants, the phononic crystals can yield complete phononic BGs [16–18]. For the second mechanism, the resonances of scattering units play a major role in the BGs which are less dependent on the periodicity and symmetry of the structure. The frequency range of the gap could be almost two orders of magnitude lower than the usual Bragg gap [19,20].

In order to promote the engineering application of PCs, the acquisition of large and tunable BGs at low frequencies is of extremely importance. Kushwaha and Djafari-Rouhani [21] computed extensive band structures for periodic arrays of rigid metallic rods in air and obtained multiple complete acoustic stop bands. Li et al. [22] investigated the effects of orientations of square rods on the acoustic band gaps in two-dimensional periodic arrays of rigid solid rods embedded in air and concluded that the acoustic band gaps can be opened and enlarged greatly by increasing the rotation angle. Cheng et al. [23] demonstrated that a class of ultrasonic metamaterial, which was composed of subwavelength resonant units, built up by parallel-coupled Helmholtz resonators with identical resonant frequency, possessed broad locally resonant forbidden bands and the bandwidths were strongly dependent on the number of resonators in each unit. Li et al. [24] studied phononic band structure with periodic elliptic inclusions for the square lattice based on the plane wave expansion method and the numerical results showed the systems composed of tungsten elliptic rods embedded in silicon matrix can exhibit a larger complete band gap than the conventional circular phononic crystal slabs. Cui et al. [25] presented a new band gap structure composed of a square array of parallel steel

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tubes with narrow slits and obtained large band gap and low starting frequency by arranging different width of slits embedded in the tubes. Lin et al. [26] presented a theoretical study on the tunability of phononic band gaps in two-dimensional phononic crystals consisting of various anisotropic cylinders in an isotropic host. Phononic band gaps for bulk acoustic waves propagating in the phononic crystal can be opened, modulated, and closed by reorienting the anisotropic cylinders. Yu et al. [27] studied the band gap properties of a two-dimensional phononic crystal with neck structure and showed that the band gaps were significantly dependent upon the geometrical parameters.

Jerusalem cross slot structures are widely used in planar microwave photonic crystal and the absorption frequency band can be flexibly adjusted by the slot parameters [28–30]. However, as far as we know, the band gap properties in phononic crystals with Jerusalem cross slot structures have not yet been carefully investigated. In this paper, we investigate the band structures in a novel two-dimensional phononic crystal composed of periodic Jerusalem cross slot in air matrix by using the finite element method (FEM) [31]. The formation mechanisms of the band gap are analyzed based on the acoustic modal analysis. Furthermore, the effects of the geometry parameters on the band gaps are discussed. Numerical results show that the band gaps can be tuned in a wide frequency range by the geometry parameters such as the slot length and width.

2. Model and methods of calculation

In this work, we consider a novel two-dimensional phononic crystal composed of periodic Jerusalem cross slot in air matrix with a square lattice. Fig. 1(a) and (b) shows the cross section of the proposed PC structure. The geometrical parameters of the Jerusalem cross slot structure are defined as follows: the parameters of the slot length are l and m respectively, and the parameters of the slot width are n and d respectively. The infinite system of the two-dimensional PC is formed by repeating the unit cell periodically along the x- and y-directions simultaneously. In the unit cell, the lattice constant $a=36$ mm, the slot length $l=28$ mm and the slot width $n=2$ mm. These three parameters remain unchanged in all the calculations below.

In order to theoretically investigate the band gap properties of the proposed PC structure, the finite element method based on the Bloch theory is applied to calculate the dispersion relations and the transmission coefficient spectra. Since the infinite system is periodic along the x- and y-directions simultaneously, only the unit cell shown in Fig. 1(b) needs to be considered. As the unit cell is composed of air and steel materials, the calculation area can be divided into the fluid and solid domains.

In the fluid domain, the governing equation of the acoustic waves can be simplified as frequency-domain Helmholtz equation:

$$\nabla\left(-\frac{1}{\rho_0}\nabla p\right)=\frac{\omega^2 p}{\rho_0 c_s} \tag{1}$$

where p is the acoustic pressure, ρ_0 is the density, ω is the angular frequency and c_s is the speed of sound.

As the acoustic impedance of air is much smaller than that of steel, one knows that the longitudinal waves propagating in air will be almost reflected by the steel inclusions and the wave propagation in the proposed PC is predominant in the air domain. So the transverse waves in steel inclusions can be neglected for the sake of simplicity, and we can consider the steel inclusions as fluid with very high stiffness and specific mass. Based on the Bloch theorem, periodic boundary conditions are applied at the boundaries between the unit cell and its four adjacent cells:

$$p(\mathbf{r}+\mathbf{a})=p(\mathbf{r})e^{i\boldsymbol{K}a} \tag{2}$$

where $\mathbf{r}$ is the position vector, $\mathbf{a}$ denotes the basis vector of the periodic structure and the parameter $\boldsymbol{K}$ is defined as a two-dimension Bloch wave vector. We solved the eigenvalue equations (Eqs. (1) and (2)) with COMSOL Multiphysics 3.5a software [32]. The Acoustic Module operating under the two-dimensional pressure acoustics Application Mode (acpr) is chosen for the calculations. The constant boundary condition is imposed on the boundary between the air and steel boards, and the Bloch boundary conditions are imposed on the two opposite boundaries of the unit cell. The unit cell is meshed by using a triangular mesh with the Lagrange quadratic elements provided. One knows that with a given value of Bloch wave vector $\boldsymbol{K}$, a group of eigenvalues and eigenmodes can be calculated by solving the eigenvalue problem. Letting the value of Bloch wave vector $\boldsymbol{K}$ along the boundary of the first Brillouin zone and repeating the calculation and we can obtain the dispersion relations of the PCs.

For the transmission spectrum, the acpr mode of COMSOL Multiphysics 3.5a software is applied to solve the transmission spectra problem. We consider a finite array structure composed of

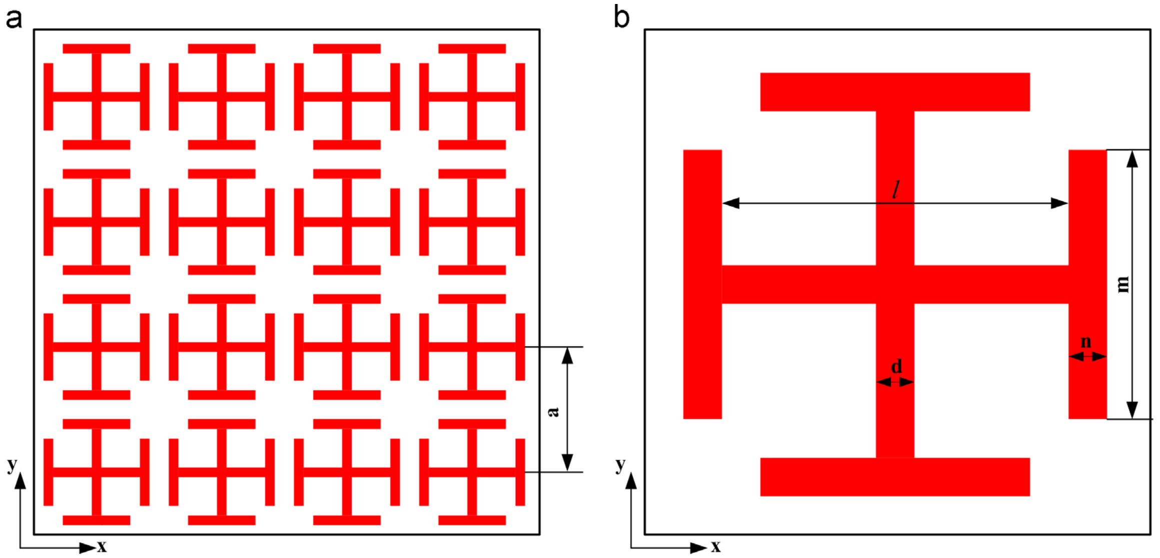

Fig. 1. (a) Schematics of the gap structure composed of a periodic square array of Jerusalem cross slot in air matrix. The lattice constant is a. (b) Schematics of the unit cell of the gap structure, in which l and m are the parameters of the slot length; n and d are the parameters of the slot width.