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Phase-flip bifurcation in a coupled Josephson junction neuron system

Kenneth Segall [a,*], Siyang Guo [a], Patrick Crotty [a], Dan Schult [b], Max Miller [a]

[a] Department of Physics and Astronomy, Colgate University, Hamilton, NY 13346, United States
[b] Department of Mathematics, Colgate University, Hamilton, NY 13346, United States

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ABSTRACT

Aiming to understand group behaviors and dynamics of neural networks, we have previously proposed the Josephson junction neuron (JJ neuron) as a fast analog model that mimics a biological neuron using superconducting Josephson junctions. In this study, we further analyze the dynamics of the JJ neuron numerically by coupling one JJ neuron to another. In this coupled system we observe a phase-flip bifurcation, where the neurons synchronize out-of-phase at weak coupling and in-phase at strong coupling. We verify this by simulation of the circuit equations and construct a bifurcation diagram for varying coupling strength using the phase response curve and spike phase difference map. The phase-flip bifurcation could be observed experimentally using standard digital superconducting circuitry.

1. Introduction

Dynamic systems such as neural networks, social networks and weather systems behave nonlinearly and often chaotically. Even though characteristics and functions of individual components in these systems are well understood, their collective behaviors remain puzzling and elusive. In the case of neural systems, billions of individual neurons make up a complex network that not only controls the entire body, but also produces consciousness, memory, knowledge, and emotions. While the behaviors of individual neurons have been well studied at the cellular, compartmental, and molecular levels during the past century, analyzing collective behaviors of neural networks is essential in moving towards a better understanding of the human brain.

Large-scale digital simulation projects on neural systems have offered extensive insights into the dynamics of complex neural networks [1]. Although these projects significantly improve efficiency by effective parallel computing designs, simulation time remains an obstacle. Analog simulations using very-large-scale-integrated circuitry [2] improve realism and speed, yet compromise on complexity and power consumption.

In a previous work [3] we showed an alternative direction for the simulation of coupled neural dynamics: using superconducting Josephson junctions to model neurons connected with real-time synaptic circuitry. Because of a superconductor's unique properties, the Josephson junction neuron (JJ neuron) is capable of simulating neural network dynamics several orders of magnitude faster than current digital or analog techniques, allowing us to learn more about neural interactions, specifically synchronization, long-term dynamics, and bifurcations. Understanding these group behaviors can lead to further studies on larger scale neural networks and brain behavior. In addition, JJ neurons have the potential to be very energy efficient [4], which would allow scaling to large networks.

Because the dynamics of a JJ neuron happen on the picosecond timescale, it will be almost impossible to observe individual action potentials in a circuit. However, collective behaviors of a system of neurons, like synchronization, can be observed in the laboratory using standard superconducting digital circuitry. Furthermore, in studying networks of neurons, collective behaviors are more informative of the system's dynamics: they are responsible for the various functionalities of neural networks such as spatial recognition, memory and sensory processes.

In this work, we show computational results for the synchronization of two mutually coupled JJ neurons. Using concepts borrowed from neuroscience [5], we compute first the phase response curve (PRC) of a single JJ neuron and then the spike phase difference map (SPDM) of the two-neuron system. With the SPDM we identify fixed points of the map where the system settles into synchronization at a fixed phase difference. We find that this phase is almost always π or 2π, meaning that the synchronization is in-phase or out-of-phase. We then construct a bifurcation diagram of these fixed points versus the coupling strength between the neurons and find a phase-flip bifurcation [6], where the system goes from in-phase synchronization at weak coupling to out-of-phase synchronization at strong coupling. We verify this behavior with simulations of the fully coupled system. This collective behavior could potentially be observed experimentally.

* Corresponding author. Tel.: +1 315 228 6597; fax: +1 315 228 7187.
E-mail address: ksegall@colgate.edu (K. Segall).

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The paper is organized as follows. Section 2 introduces the circuit model for two mutually coupled neurons, shows the equations of motion that describe their dynamics, and discusses the calculation of the PRC for a single JJ neuron and the SPDM for the coupled system. Section 3 discusses the bifurcation diagram and the appearance of a phase-flip bifurcation. Section 4 discusses the results and the possibility to detect this bifurcation experimentally.

2. Model and method

Fig. 1 shows the system of mutually coupled Josephson junction neurons. Josephson junctions are labeled as $\times$'s in the diagram. Each half of the circuit has a soma (JJ neuron), a Josephson Transmission Line (JTL) as an axon, and a synapse. Coupling resistors connect the two halves. The parameters of both halves of the circuit are identical.

The JJ neuron model uses three inductors and two Josephson junctions to mimic neural action potentials [3]. Our previous work showed that the JJ neuron model mimics basic nerual characteristics such as an action potential with a distinctive shape, a threshold potential, and a refractory period [3]. A biological neuron controls its membrane potential primarily by the presence of sodium and potassium ions. The in- and out-flux of these ions, governed by diffusion, electric potentials, and ion channels, produces changes in the membrane potential, and can produce action potentials [7]. The two Josephson junctions in the JJ neuron, referred to as the pulse junction (p) and control junction (c), mimic the flow of sodium and potassium ions in a biological neuron, respectively. The bias current (I_b) provides energy to the circuit. The addition of a JTL as a model axon allows spatial propagation of the action potential, making the JJ neuron structurally analogous to a typical biological neuron. The JTL [8] is a line of single Josephson junctions which can be connected to a single JJ neuron. It preserves the shape and velocity of action potentials initiated by the JJ neuron. Using a JTL is advantageous for the future fabrication of such circuits in mass quantities, since it is one of the major components of superconducting digital circuitry [9] and there exist industrial designs for JTLs. We can adjust the length of the axon by modifying the number of junctions (N) in the JTL.

The equations of motion for the coupled circuit, similar to that for a single JJ neuron [2], come from Kirchhoff's Laws and fluxoid quantization. The equation of motion for the phase of the jth junction ($j \neq 1$ or N) of the JTL is

$$\ddot{\phi}_j = -\Gamma_{JTL}\dot{\phi}_j - \sin\phi_j + \eta_{JTL} i_{JTL} + \eta_{JTL}\lambda_{JTL}(\phi_{j+1}+\phi_{j-1}-2\phi_j) \tag{1}$$

The equation of motion for the phase of the first junction (which is actually the pulse junction of the JJ neuron) and the Nth junction of the JTL are

$$\ddot{\phi}_1 = -\Gamma\dot{\phi}_1 - \sin\phi_1 - \lambda(\phi_c+\phi_1) + \Lambda_s(i_{\text{in}}+i_{21}) + (1-\Lambda_p)i_b - 2(1-\Lambda_p)\eta_{\text{JTL}}\lambda_{\text{JTL}}(\phi_1-\phi_2) \tag{2}$$

$$\ddot{\phi}_N = -\Gamma_{\text{JTL}}\dot{\phi}_N - \sin\phi_N + 2\eta_{\text{JTL}}\lambda_{\text{JTL}}(\phi_{N-1}-\phi_N) + \eta_{\text{JTL}} i_{\text{JTL}} - i_{12} - (\lambda/\Lambda_{\text{syn}}\omega_0^2)V_{\text{out1}} \tag{3}$$

The equation for the phase of the control junction in the JJ neuron is

$$\ddot{\phi}_c = -\Gamma\dot{\phi}_c - \sin\phi_c - \lambda(\phi_c+\phi_1) + \Lambda_s(i_{\text{in}}+i_{21}) - \Lambda_p i_b + 2\Lambda_p\eta_{\text{JTL}}\lambda_{\text{JTL}}(\phi_1-\phi_2) \tag{4}$$

The equations for the voltage and current output from the synapse are

$$\ddot{V}_{\text{out1}} = \omega_0^2\left[\dot{\phi}_N - V_{\text{out1}} - (\Gamma_{\text{syn}}/\lambda)\dot{i}_{12} - (Q\omega_0\Lambda_{\text{syn}}/\lambda)i_{12} - (Q/\omega_0)\dot{V}_{\text{out1}}\right] \tag{5}$$

$$\dot{i}_{12} = \frac{\lambda\left[V_{\text{out1}} - r_{12}i_{12}/\Gamma - \Lambda_{\text{syn}}(\dot{\phi}_{N+1}+\dot{\phi}_{c2})\right]}{\Lambda_s(1-\Lambda_s)} \tag{6}$$

The equations of motion for the other half of the circuit are identical to those of the first half except for changing appropriate variables and indexes for current, voltage, and phase. Descriptions of the parameters used in and default values are given in Table 1. They are chosen to give continuously spiking action potentials. The key parameters of interest are the coupling resistances, R_{12} and R_{21}, which set the strength of the coupling. In our simulations they were always equal to each other and will be just referred to as the "coupling resistance". Eqs. (1)–(6) were numerically integrated with a Runge–Kutta algorithm to compute the PRC and to verify the phase-flip bifurcation.

In their 2003 paper [5], Acker, Kopell, and White described a method of mapping out synchronization dynamics of a coupled system by looking at how individual neurons respond to stimuli. In a coupled system, a spike time difference map (STDM) can represent the dynamics of the pair. A STDM gives the change in

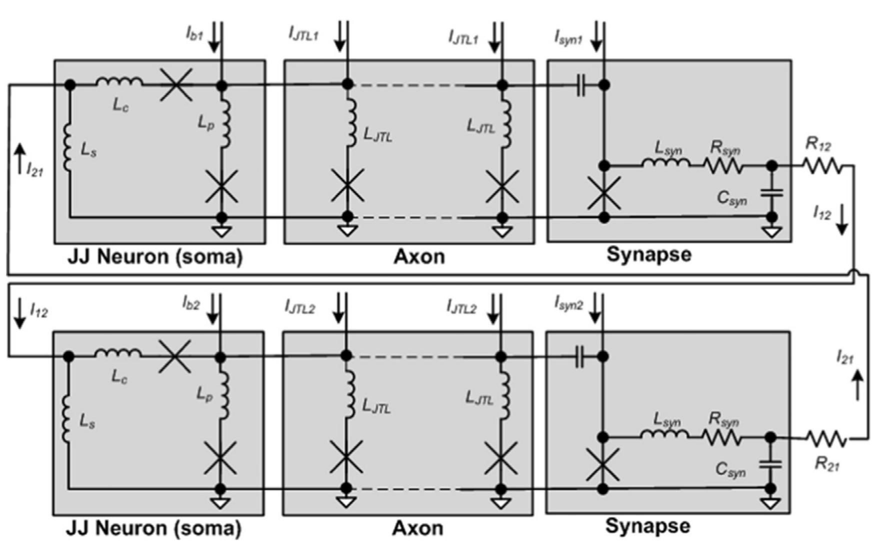

Fig. 1. Schematic of two coupled Josephson junction neurons. Josephson junctions are indicated by an "$\times$". The soma (a single JJ neuron) connects to the synapse through an axon, which is a Josephson Transmission line. Coupling resistors (R_{21} and R_{12}) connect the two halves of the circuit.