Physica B 452 (2014) 74–81

Contents lists available at ScienceDirect

Physica B

journal homepage: www.elsevier.com/locate/physb

Generation of extended states in diluted transmission lines with distribution of inductances according to Galois sequences: Hamiltonian map approach

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ARTICLE INFO

Article history:
Received 10 June 2014
Accepted 6 July 2014
Available online 15 July 2014

Keywords:
Localization properties
Galois sequences
Diluted aperiodic transmission lines
Hamiltonian map

ABSTRACT

We study the localization properties of diluted direct transmission lines, when we distribute two values of inductances L_A and L_B, according to the aperiodic Galois sequence. When we dilute the aperiodic Galois system with $(d-1)$ inductances with constant L_0 value, we find d sub-bands and $(d-1)$ gaps; here d is the period of the distribution of the Galois sequence in the diluted system. Under the condition $L_0 \approx (L_A, L_B)$, we find a set of extended states for finite N_d system size, which disappears when $N_d \to \infty$. For the case $L_0 \gg (L_A, L_B)$, using the scaling behavior of the averaged participation number $\langle D(\omega) \rangle$ and the scaling behavior of the averaged normalized participation number $\langle \xi(\omega) \rangle$, we demonstrate the existence of extended states in the thermodynamic limit.

1. Introduction

The localization properties of aperiodic, quasi-periodic and disordered classical and quantum systems have been studied experimentally and theoretically [1–18]. The short-range and long-range correlations are very important properties which determine the possible existence of resonances (a finite number of extended states) or the possible existence of extended states' bands, respectively. For one-dimensional systems without correlation in the disorder (white noise), all states are localized states in the thermodynamic limit $(N \to \infty)$. The study of aperiodic and quasi-periodic systems reveals rich electronic properties [19–29]. In addition, in the last time, the localization properties of direct and dual electric transmission lines (TL) have been studied [30–36]. These classical systems have been studied considering aperiodic, quasi-periodic and long-range correlated distribution of L_j inductances and C_j capacitances.

Also, diluted disordered classical systems (harmonic oscillators and transmission lines) and diluted disordered tight-binding quantum systems have been studied in the last time [12–14,30,31,37–45].

Recently, the localization properties of electronic states of one-dimensional finite aperiodic Galois sequences have been studied [46]. Using a finite sequence with $N = 1023$ sites $(m=10)$, the authors demonstrate that all states are localized states. In the present work we study the localization properties of non-diluted and diluted classical electrical direct transmission lines, when we distribute two different values of inductances L_A and L_B according to the Galois sequence [46–53]. We study the finite Galois sequences generated for specific p and m. The N number of elements of each aperiodic Galois sequence is given by the relation $N=(p^m-1)$. In this paper we use $p=2$, with m varying from $m=13$ to $m=23$, namely, the N size of each Galois sequence varies from $N=8191$ to $N=8{,}388{,}607$.

For the diluted case, we introduce a set of $(d-1)$ diluting inductances of fixed L_0 value between two consecutive sites with Galois elements. When the Galois aperiodic sequence with $N=(2^m-1)$ elements is diluted using $(d-1)$ constant L_0 values between two consecutive Galois elements, the N_d total number of cells of the diluted transmission line is given by $N_d=(d(2^m-2)+1)$. For the case $d=5$, N_d ranges from $N_d=40{,}951$ $(m=13)$ to $N_d=41{,}943{,}031$ $(m=23)$.

In this paper, we calculate the global density of states $DOS(\omega)$ and the integrated density of states $IDOS(\omega)$ using the Dean method. Besides, we use the Hamiltonian map approach [18,54–61] to calculate the electric current function $I(\omega)$, the normalized localization length $\Lambda(\omega)$, the transmission coefficient $T(\omega)$, the participation number $D(\omega)$ and the normalized participation number $\xi(\omega)=(1/N)D(\omega)$, where N is the system size under study. We analyze the localization properties in the thermodynamics limit studying the scaling behavior of the participation number $D(\omega)$ and the scaling behavior of the normalized participation number $\xi(\omega)$.

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http://dx.doi.org/10.1016/j.physb.2014.07.009
0921-4526/

The primary results of our study of the localization properties of the diluted TL with distribution of inductances following the Galois sequence with period d and $(d-1)$ diluting L_0 values, are the following:

(a) For $L_0 \approx (L_A, L_B)$, for finite N_d diluted system size, each sub-band contains a set of extended states $(\Lambda(\omega, N_d) \geq 1$ and $T(\omega, N_d)\ldots 1)$, which begin to localize for increasing N_d values. As a consequence, for this case, all states are localized states in the thermodynamic limit $(N_d \to \infty)$.

(b) For finite N_d and $L_0 \gg (L_A, L_B)$, each sub-band contains a set of extended states $(\Lambda(\omega, N_d) \geq 1$ and $T(\omega, N_d)\ldots 1)$. Using the scaling properties of averaged quantities $\langle D(\omega, N_d)\rangle$ and $\langle \xi(\omega, N_d)\rangle$, we demonstrate the existence of extended states in the thermodynamic limit $(m \to \infty,\ N_d \to \infty)$. This is one of the most important results of our work.

This paper is organized as follows: Section 2 describes the model and the method. Section 3 shows the most important numerical results and Section 4 provides the conclusions of our work.

2. Model and method

2.1. Direct electrical transmission lines

The dynamic equation for the direct diagonal transmission lines formed by horizontal inductances L_n and constant vertical capacitances $C_n = C_0, \forall n$ is given by Refs. [30,31]

$$(2-\omega^2 C_0 L_n) I_n - I_{n-1} - I_{n+1} = 0 \tag{1}$$

where ω is the frequency. In this paper, we distribute two different values of inductances, L_A and L_B, using the Galois aperiodic sequence [46–48,50–53]. It is interesting to note that the dynamic equation (1) can be mapped into a tight-binding quantum model [30]. In this paper we will use the Hamiltonian map approach to solve the infinite set of equations given by Eq. (1).

2.2. Generation of Galois sequences

The finite fields or Galois fields $GF(p^m)$ are very important in informatics and communication, because they are basic for the study of decoding theory and cryptography. Additionally, they are essential in the study of discrete mathematics [47–53]. The Galois fields are correctly defined when p is a prime number and m is a positive integer. In this paper, the Galois sequences $\{a_k\}$ with period (p^m-1) are generated from the primitive polynomial $P_m(x)$ (see Table 1) using the recursion method [47,48,51,52]. We will work with a TL with an aperiodic distribution of inductances with an N number of elements coincident with the period, namely, $N=(p^m-1)$. We will study the case with $p=2$, which means that the elements of the Galois field can have only two different values, 0 and 1. In Table 1 we show the primitive polynomial $P_m(x)$ and the corresponding $\{a_{k+m}\}$ recurrence relation, necessary to generate the Galois sequence, for $m=15$ to $m=22$.

Table 1
Primitive polynomials $P_m(x)$ and the corresponding recurrence relations a_{k+m}, for $k \in [1, (p^m-1-m)]$ to obtain the Galois sequence. The initial conditions for the first m values are considered.

m	$P_m(x)$	a_{k+m}, $k \geq 1$
15	$x^{15}+x+1$	$a_{k+1}+a_k$
16	$x^{16}+x^5+x^3+x^2+1$	$a_{k+5}+a_{k+3}+a_{k+2}+a_k$
17	$x^{17}+x^3+1$	$a_{k+3}+a_k$
18	$x^{18}+x^7+1$	$a_{k+7}+a_k$
19	$x^{19}+x^6+x^5+x+1$	$a_{k+6}+a_{k+5}+a_{k+1}+a_k$
20	$x^{20}+x^3+1$	$a_{k+3}+a_k$
21	$x^{21}+x^2+1$	$a_{k+2}+a_k$
22	$x^{22}+x+1$	$a_{k+1}+a_k$

We use the same arbitrary initial conditions of Ref. [46], namely, $a_{(2n-1)}=1$ and $a_{(2n)}=0$, for $1 \leq n \leq [m/2]$, where $[x]$ means the integer part of the real number x. These initial conditions are already considered in Table 1, for the first m values of each sequence $(a_1$–$a_m)$. As a consequence, we will work with aperiodic Galois sequences defined between 1 and (p^m-1), which means that the recurrence relation, $\{a_{k+m}\}$ given in Table 1, will be defined for $k \in [1, (p^m-1-m)]$. To work with two different values of the inductances, i.e., L_A and L_B, distributed according to the aperiodic Galois sequence, we make the following correspondence: $1 \to L_A$ and $0 \to L_B$. We show an example for the aperiodic case with $m=4$ $(N=15)$:

$L_A L_B L_A L_B L_A L_A L_A L_A L_B L_B L_B L_A L_B L_B L_A$

2.3. Hamiltonian map approach

Using the substitution $\alpha_n = (\omega^2 C_0 L_n)$, the dynamic equation (1) can be written as

$$(2-\alpha_n) I_n = I_{n-1} + I_{n+1} \tag{2}$$

We will study the localization properties of the disordered TL using the Hamiltonian map approach [18,54–61]. To generate a classical two-dimensional Hamiltonian map corresponding to the direct electric transmission line, let us consider the following definition of two new variables x_n and p_n as a function of the electric current functions I_n and I_{n+1}:

$$x_n = I_n \tag{3}$$

$$p_{n+1} = I_{n+1} - I_n \tag{4}$$

After some algebra, we find the Hamiltonian map for our problem, namely,

$$\begin{aligned} x_{n+1} &= \beta_n x_n + p_n \\ p_{n+1} &= -\alpha_n x_n + p_n \end{aligned} \tag{5}$$

where $\beta_n = (1-\alpha_n)$. The trajectories of this map in the plane (p, x) can be used to recognize the localized or extended character of the electric current function $I_n = x_n$. In particular, if all inductances are constant, i.e., $L_n = L_0$, $\forall n$, the transmission line is periodic, and all trajectories of the map in the phase space (p, x) are circles specified for the initial conditions (p_0, x_0). As a consequence, the extended states are represented by bounded trajectories; on the contrary, the localized states are represented by unbounded trajectories. In addition, the study of the time evolution of the Hamiltonian map (5) is similar to the transfer matrix method used in the study of disordered systems [18,62].

Let us now consider the transformation of the map (5) to the canonical variables (r, θ) in the usual way, i.e.,

$$x = r \sin\theta \tag{6}$$

$$p = r \cos\theta \tag{7}$$

The Hamiltonian map (5) can be written in the following form:

$$r_{n+1} \sin\theta_{n+1} = \beta_n r_n \sin\theta_n + r_n \cos\theta_n \tag{8}$$

$$r_{n+1} \cos\theta_{n+1} = -\alpha_n r_n \sin\theta_n + r_n \cos\theta_n \tag{9}$$

using these equations, we can calculate the term $\Gamma_n = ((r_{n+1})/r_n)$, namely,

$$\Gamma_n^2 = 2-(1+2\alpha_n\beta_n)\sin^2\theta_n + (1-2\alpha_n)\sin 2\theta_n \tag{10}$$

Furthermore, dividing Eq. (8) by Eq. (9), we obtain the recurrence equation for the phase map θ_n, namely, the relation between θ_{n+1}