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Quantum criticality of geometric phase in coupled optical cavity arrays under linear quench

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ABSTRACT

The atoms trapped in microcavities and interacting through the exchange of virtual photons can be modeled as an anisotropic Heisenberg spin-1/2 lattice. We study the dynamics of the geometric phase of this system under the linear quenching process of laser field detuning, which shows XX criticality of the geometric phase and also gives the result of quantum criticality for different quenching rates.

1. Introduction

The recent experimental success in engineering strong interaction between photons and atoms in high quality micro-cavities open up the possibility to use light matter system as quantum simulators for many body physics [1–16]. The authors of Refs. [2–4] have shown that an effective spin lattice can be generated with individual atoms in micro-cavities that are coupled to each other via exchange of virtual photons. The two states of spin polarization are represented by two long lived atomic levels in the system.

Many body Hamiltonians can be created and probed in coupled cavity arrays. In our previous study, we have explained the physics of arrays formed with micro-optical cavities [4]. The atoms in the cavity are used for detection and also for the generation of interaction between photons in the same cavity. As the distance between the adjacent cavities is considerably larger than the optical wave length of the resonant mode, individual cavities can be addressed. This artificial system can act as a quantum simulator. There are quite a few experimental studies of quantum simulation using light in cavities containing a qubit [17,18]. This micro-cavity system shows different quantum phases and quantum phase transitions [4,19].

To the best of our knowledge, here we not only study the dynamics of geometric phase but also solve the nature of criticality under a laser field detuning quenching process for different quenching rates, which has not been addressed previously in the literature of the cavity QED system [1–16].

Here we mention very briefly the essence of the geometric phase in the condensed matter physics. The geometric phases have been associated with a variety of condensed matter phenomena [20–24]. Besides various theoretical investigations, geometric phases have been experimentally tested in various cases, *e.g.* with photons [25–27], with neutrons [28,29] and with atoms [30]. The quantum state engineering of cavity QED has advanced in recent times due to the rapid experimental/technological progress [31–35]. We hope that the theoretical scheme which we propose for the laser field detuning induce quenching process in the dynamics of geometric phase will be observed experimentally in photonic systems. In our study, we consider the three level system in the cavity with radiation field and applied externally laser. There are few experimental studies with three level system in the presence of radiation [31–35].

In this paragraph we discuss very briefly the dynamics of geometric phase and its relation in quantum phase transition [19]. The generation of the geometric phase (GP) as a witness of a singular point in an energy spectrum arises in all non-trivial geometric evolutions. In this respect, the connection of the geometric phase with quantum phase transition (QPTs) has been explored very recently [36–39]. Since response times typically diverge in the vicinity of critical point, sweeping the phase transition with a finite velocity leads to a breakdown of adiabatic condition and generates interesting dynamical (non-equilibrium) effects. In the case of thermal phase transitions, the Kibble–Zurek (KZ) mechanism [41,42] explains the formation of defects via rapid

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cooling. This idea of defect formation in the second order phase transition has been extended to zero temperature QPTs [43,44] by studying the spin models under linear quench. This is basic physics of our present study in a cavity QED lattice under the quenching of laser field detuning. The evolution of geometric phase is realized through the adiabatic process. In the next section we will discuss in detail how to restore the criteria of adiabaticity during the linear quenching process.

The micro-cavities of a photonic crystal are coupled through the exchange of photons. Each cavity consists of one atom with three levels in the energy spectrum, two of them are long lived and represent two spin states of the system and the other represents excited state [2–4]. Externally applied laser and cavity modes couple to each atom of the cavity. It may induce the Raman transition between these two long-lived energy levels. Under a suitable detuning between the laser and cavity modes, virtual photons are created in the cavity, which mediate interactions with another atom in a neighboring cavity. One can eliminate the excited states by choosing the appropriate detuning between the applied laser and cavity modes. One can achieve only two states per atom in the long lived state and the system can be described by a spin-1/2 Hamiltonian [2–4].

The Hamiltonian of our present study consists of three parts:

$$H = H_A + H_C + H_{AC}. \tag{1}$$

The Hamiltonians are the following:

$$H_A = \sum_{j=1}^{N} \omega_e |e_j\rangle\langle e_j| + \omega_{ab} |b_j\rangle\langle b_j|, \tag{2}$$

where j is the cavity index, ω_{ab} and ω_e are the energies of the state $|b\rangle$ and the excited state respectively. The energy level of state $|a\rangle$ is set as zero, and $|a\rangle$ and $|b\rangle$ are the two stable states of an atom in the cavity and $|e\rangle$ is the excited state of that atom in the same cavity. The following Hamiltonian describes the photons in the cavity:

$$H_C = \omega_C \sum_{j=1}^{N} c_j^\dagger c_j + J_C \sum_{j=1}^{N} (c_j^\dagger c_{j+1} + h.c), \tag{3}$$

where $c_j^\dagger(c_j)$ is the photon creation (annihilation) operator for the photon field in the jth cavity, ω_C is the energy of photons and J_C is the tunneling rate of photons between neighboring cavities. The atom–photon interaction and the coupling with lasers are described by

$$H_{AC} = \sum_{j=1}^{N} \left[\left(\frac{\Omega_a}{2} e^{-i\omega_a t} + g_a c_j \right) |e_j\rangle\langle a_j| + h.c. \right] + [a \leftrightarrow b]. \tag{4}$$

Here g_a and g_b are the couplings of the cavity mode for the transition from the energy states $|a\rangle$ and $|b\rangle$ to the excited state. Ω_a and Ω_b are the Rabi frequencies of the lasers with frequencies ω_a and ω_b respectively.

The authors of Refs. [2–4] have derived an effective spin model by considering the following physical processes: a virtual process regarding emission and absorption of photons between two stable states of neighboring cavity yields the resulting effective Hamiltonian as

$$H_{xy} = \sum_{j=1}^{N} B\sigma_j^z + \sum_{j=1}^{N} \left(\frac{J_1}{2} \sigma_j^\dagger \sigma_{j+1}^- + \frac{J_2}{2} \sigma_j^- \sigma_{j+1}^- + h.c. \right). \tag{5}$$

When J_2 is real then this Hamiltonian reduces to the XY model. Where $\sigma_j^z = |b_j\rangle\langle b_j| - |a_j\rangle\langle a_j|$, $\sigma_j^+ = |b_j\rangle\langle a_j|$, $\sigma_j^- = |a_j\rangle\langle b_j|$

$$H_{xy} = \sum_{i=1}^{N} [B\sigma_i^z + J_1(\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y] + J_2(\sigma_i^x \sigma_{i+1}^x - \sigma_i^y \sigma_{i+1}^y))$$
$$= \sum_{i=1}^{N} B(\sigma_i^z + J_x \sigma_i^x \sigma_{i+1}^x + J_y \sigma_i^y \sigma_{i+1}^y) \tag{6}$$

with $J_x = (J_1 + J_2)$ and $J_y = (J_1 - J_2)$.

We follow Refs. [2,45] to present the analytical expression for the different physical parameters of the system

$$B = \frac{\delta_1}{2} - \beta, \tag{7}$$

$$\beta = \frac{1}{2} \left[\frac{|\Omega_b|^2}{4\Delta_b} \left(\Delta_b - \frac{|\Omega_b|^2}{4\Delta_b} - \frac{|\Omega_b|^2}{4(\Delta_a - \Delta_b)} - \gamma_b g_b^2 - \gamma_1 g_a^2 + \gamma_1^2 \frac{g_a^4}{\Delta_b} \right) - (a \leftrightarrow b) \right], \tag{8}$$

$$J_1 = \frac{\gamma_2}{4} \left(\frac{|\Omega_a|^2 g_b^2}{\Delta_a^2} + \frac{|\Omega_b|^2 g_a^2}{\Delta_b^2} \right), \tag{9}$$

$$J_2 = \frac{\gamma_2}{2} \left(\frac{\Omega_a \Omega_b g_a g_b}{\Delta_a \Delta_b} \right). \tag{10}$$

where $\gamma_{a,b} = (1/N)\sum_k 1/(\omega_{a,b} - \omega_k)$, $\gamma_1 = (1/N)\sum_k 1/((\omega_a + \omega_b)/2 - \omega_k)$, and $\gamma_2 = (1/N)\sum_k e^{ik}/((\omega_a + \omega_b)/2 - \omega_k)$, $\delta_1 = \omega_{ab} - (\omega_a - \omega_b)/2$, $\Delta_a = \omega_e - \omega_a$, $\Delta_b = \omega_e - \omega_a - (\omega_{ab} - \delta_1)$, $\delta_a^k = \omega_e - \omega_k$, $\delta_b^k = \omega_e - \omega_k - (\omega_{ab} - \delta_1)$, g_a and g_b are the couplings of respective transition to the cavity mode, Ω_a and Ω_b are the Rabi frequency of laser with frequencies ω_a and ω_b, respectively.

2. Model hamiltonian and quantum phases

We express our model Hamiltonian in the following form:

$$H = \sum_n [(1+\alpha) S_n^x S_{n+1}^x + (1-\alpha) S_n^y S_{n+1}^y + B \sum_n S_n^z], \tag{11}$$

where S_n^α are the spin-1/2 operators. We assume that the XY anisotropy $0 < \alpha \le 1$ and the magnetic field strength is $h \ge 0$. The parameters correspondence between the micro-cavities and spin chain are the following: $J_1 = 1$ and $J_2 = \alpha$. Here, we calculate the geometric phase and its dynamics under the quenching of magnetic field. In this model, the geometric phase of the ground state is evaluated by applying a rotation of ϕ around the z-axis in a closed circuit to each spin [36,46,47]. A new set of Hamiltonian H_ϕ is constructed from the Hamiltonian (H) as

$$H_\phi = U(\phi) H U^\dagger(\phi), \tag{12}$$

where $U(\phi) = \prod_{j=-M}^{+M} \exp(i\phi\sigma_j^z/2)$, and σ_j^z is the z component of the standard Pauli matrix at site j. Here M is the integer which relates with the lattice site numbers by the following relation: $2M+1 = N$. The family of Hamiltonians generated by varying ϕ has the same energy spectrum as the initial Hamiltonian and $H(\phi)$ is π-periodic in ϕ. We use the Jordan–Wigner transformation to convert the spin chain system to the one-dimensional spinless fermions system. We use the following analytical relation: $a_j = (\prod_{i<j} \sigma_i^z) \sigma_j^\dagger$ and then use the Fourier transforms of the fermionic operator, $d_k = (1/\sqrt{N}) \sum_{j=1}^{N} a_j \exp(-2\pi j_k/N)$ with $k = -M, \ldots, +M$. The Hamiltonian H_ϕ can be diagonalized by transforming the fermionic operators in the momentum space and then using the Bogoliubov transformation. The ground state $|g\rangle$ of the system is expressed as [36–39]

$$|g\rangle = \prod_{k>0} \left(\cos\frac{\theta_k}{2} |0\rangle_k |0\rangle_{-k} - i\exp(2i\phi) \sin\frac{\theta_k}{2} |1\rangle_k |1\rangle_{-k} \right), \tag{13}$$

where $|0\rangle_k$ and $|1\rangle_k$ are the vacuum and single fermionic excitation of the k-th momentum mode respectively. The angle θ_k is given by

$$\cos\theta_k = \frac{\cos k - B}{\Lambda_k}, \tag{14}$$

and $\Lambda_k = \sqrt{(\cos k - B)^2 + \alpha^2 \sin^2 k}$ is the energy gap above the ground state. The ground state is a direct product of N spins, each lying in the two-dimensional Hilbert space spanned by $|0\rangle_k |0\rangle_{-k}$ and $|1\rangle_k |1\rangle_{-k}$. For each value of k, the state in each of the two-