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Thermodynamics of ferromagnetic spin chains in a magnetic field: Impact of the spin–wave interaction

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ABSTRACT

The thermodynamic properties of ferromagnetic spin chains have been the subject of many publications. Still, the problem of how the spin–wave interaction manifest itself in these low-temperature series has been neglected. Using the method of effective Lagrangians, we explicitly evaluate the partition function of ferromagnetic spin chains at low temperatures and in the presence of a magnetic field up to three loops in the perturbative expansion where the spin–wave interaction sets in. We discuss in detail the renormalization and the numerical evaluation of a particular three-loop graph and derive the low-temperature series for the free energy density, energy density, heat capacity, entropy density, as well as the magnetization and the susceptibility. In the low-temperature expansion for the free energy density, the spin–wave interaction starts manifesting itself at order $T^{5/2}$. In the pressure, the coefficient of the $T^{5/2}$-term is positive, indicating that the spin–wave interaction is repulsive. While it is straightforward to go up to three-loop order in the effective loop expansion, the analogous calculation on the basis of conventional condensed matter methods, such as spin–wave theory, appears to be beyond reach.

1. Introduction

In the present study we rigorously answer the question of how the spin–wave interaction manifests itself in the low-temperature properties of ferromagnetic spin chains in a weak magnetic field. Using the systematic method of effective Lagrangians, in the very recent article [1], it was argued that the spin–wave interaction only starts showing up at the three-loop level. However, the explicit evaluation of the various Feynman graphs contributing at this order to the partition function has not been addressed in that reference. This quite elaborate task is the subject of the present paper. We then provide the low-temperature series for the free energy density, energy density, heat capacity, entropy density, as well as the magnetization and the susceptibility.

The effective Lagrangian method relies on the fact that the low-energy dynamics of the system is captured by the Goldstone bosons, which result from the spontaneously broken global symmetry. In the present case, the spin rotation symmetry of the Heisenberg ferromagnet is spontaneously broken, $O(3) \to O(2)$, and the spin–waves or magnons emerge as Goldstone bosons. Conceptually, it is quite remarkable that the effective Lagrangian method works in one spatial dimension. It is well-known that in a Lorentz-invariant framework, where the Goldstone bosons (pions, kaons, η-particle) follow a linear, i.e. relativistic, dispersion relation, the method fails in one spatial dimension. However, the ferromagnet, where the spin waves obey a quadratic dispersion law, is quite peculiar: here the systematic loop expansion perfectly works as we explain below.

In the low-temperature expansion of the free energy density, the spin–wave interaction generates a term of order $T^{5/2}$. The general structure of this series is discussed, and the question of which contributions are due to free magnon particles and which ones are due to the spin–wave interaction is thoroughly answered. In view of the nonperturbatively generated energy gap, we also critically examine the range of validity of the effective low-temperature series, pointing out that it is not legitimate to take the limit of a zero magnetic field.

The thermodynamic properties of ferromagnetic spin chains have attracted a lot of attention over the past few decades and many methods have been used to study these interesting one-dimensional systems. While early investigations were based on the Bethe ansatz [2–10], the modified spin–wave theory was the method advocated in Refs. [11–13]. Further methods used to address ferromagnetic spin chains include Schwinger-boson mean-field theory [14,15], Green functions [16,18,17,19–23], variants of spin-wave theory [24], scaling methods [25,17,26–31], numerical simulations [32–38,22], and yet other approaches [39–44]. Given this abundant literature on ferromagnetic spin chains, it is really surprising that the effect of the spin–wave interaction has been

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largely neglected. In particular, although ferromagnetic spin chains can be solved exactly by e.g. the Bethe ansatz, the low-temperature series derived from these exact results all refer to either a tiny or a zero magnetic field, which does not cover the domain we are interested in here.

We emphasize that in the problem under consideration, the effective field theory approach is more efficient than conventional condensed matter methods such as spin–wave theory, as it allows one to *systematically* go to higher orders in the low-temperature expansion – beyond the results provided in the literature. Above all – for the first time, to the best of our knowledge – the manifestation of the spin–wave interaction in the low-temperature behavior of ferromagnetic spin chains in a magnetic field is discussed in a systematic manner. Almost all previous theoretical studies that analyzed the structure of the low-temperature series for ferromagnetic spin chains were restricted to the idealized picture of the free magnon gas. One exception is Ref. [24] which, however, refers to a tiny magnetic field and appears to be not quite consistent, as we point out in Section 5.

The rest of the paper is organized as follows. In Section 2 we provide the reader with some basic aspects of the effective Lagrangian technique. The low-temperature expansion of the partition function up to three-loop order is derived in Section 3. The nontrivial part concerns the renormalization of a particular three-loop graph which is discussed in detail in Section 4. The low-temperature series for the free energy density, pressure, energy density, entropy density, heat capacity, as well as the magnetization and the susceptibility for ferromagnetic spin chains in a magnetic field are given in Section 5. While our conclusions are presented in Section 6, details on the numerical evaluation of a specific three-loop graph are discussed in two appendices.

The model-independent and systematic effective Lagrangian method, unfortunately, is still not very well known among condensed matter physicists. We would like to convince the reader that this method indeed represents an alternative and rigorous theoretical framework to address condensed matter systems, by providing a list of articles which are also based on this method. Ferromagnets and antiferromagnets in three and two space dimensions were considered in Refs. [45–61]. Two-dimensional antiferromagnets, doped with either holes or electrons, which represent the precursors of high-temperature superconductors were analyzed in Refs. [62–71]. Moreover, it was demonstrated in Refs. [72–76] that the effective Lagrangian technique is perfectly consistent with both numerical simulations based on the loop-cluster algorithm and an analytically solvable microscopic model in one spatial dimension.

2. Effective Lagrangian method

In a very recent article, Ref. [1], the low-temperature expansion of partition function for the ferromagnetic spin chain in a weak magnetic field was evaluated up to two loops. Here we perform the analysis up to three-loop order, where the spin–wave interaction comes into play. Essential aspects of the effective Lagrangian method at finite temperature have been discussed in Section 2 of Ref. [1] and will not be repeated here in detail. Below, we just focus on some basic ingredients of the method. Although Section 2 of Ref. [1] is self-contained and contains all the necessary information to understand the present calculation, the interested reader may still find more details on finite-temperature effective Lagrangians in Appendix A of Ref. [48] and in the various references given therein.

The systematic construction of the effective field theory is based on an inspection of the symmetries inherent in the underlying theory. In the present case, the effective Lagrangian, or more precisely the effective action

$$\mathcal{S}_{eff} = \int d^2x \, \mathcal{L}_{eff} \tag{1}$$

describing the ferromagnetic spin chain, must share all the symmetries of the underlying Heisenberg model. These include the spontaneously broken O(3) spin rotation symmetry (at zero temperature), parity and time reversal. Note that here we refer to zero external field. As discussed below, the magnetic field is incorporated into the effective Lagrangian as a perturbation that explicitly breaks O(3). One also has to identify the relevant low-energy degrees of freedom entering the effective description. In the case of the Heisenberg ferromagnet, these are the two real magnon fields – or the physical magnon particle – that arise due to the spontaneously broken spin symmetry $O(3) \to O(2)$.

The various terms in the effective Lagrangian are organized systematically according to the number of space and time derivatives which act on the magnon fields. At low energies or temperatures, terms which contain only a few derivatives are the dominant ones, while terms with a larger number of derivatives are suppressed [77–79]. The effective Lagrangian $\mathcal{L}_{eff}$ thus amounts to a systematic derivative expansion, or, equivalently, an expansion in powers of energy and momentum. Hence the quantities of physical interest (partition function, free energy density, magnetization, etc.) derived from $\mathcal{L}_{eff}$ also correspond to expansions in powers of momentum which – at finite temperature – translate into expansions in powers of temperature.

The leading-order effective Lagrangian for the one-dimensional ferromagnet is of momentum order p^2 and reads [50]

$$\mathcal{L}^2_{eff} = \Sigma \frac{\epsilon_{ab} \partial_0 U^a U^b}{1+U^3} + \Sigma \mu H U^3 - \frac{1}{2} F^2 \partial_{x_1} U^i \partial_{x_1} U^i. \tag{2}$$

The fundamental object is the three-dimensional magnetization unit vector $U^i = (U^a, U^3)$, where the two real components $U^a (a=1,2)$ describe the spin–wave degrees of freedom. The quantity H is the magnetic field which points to the third direction, $\vec{H} = (0,0,H)$ with $H = |\vec{H}| > 0$. While the derivative structure of the above terms is determined by the symmetries of the underlying theory, the two a priori unknown low-energy coupling constants – the spontaneous magnetization at zero temperature Σ, and the constant F – have to be fixed experimentally, in a numerical simulation or by comparison with the microscopic theory. It is important to point out that one time derivative (∂_0) counts as two space derivatives $(\partial_{x_1} \partial_{x_1})$, i.e., two powers of momentum are on the same footing as one power of energy or temperature: $k^2 \propto \omega, \; T$. This is characteristic of ferromagnetic systems where the magnons display a quadratic dispersion relation.

The next-to-leading-order effective Lagrangian for the ferromagnetic spin chain is of order p^4 and involves the two effective coupling constants l_1 and l_3 [1]:

$$\mathcal{L}^4_{eff} = l_1 (\partial_{x_1} U^i \partial_{x_1} U^i)^2 + l_3 \partial^2_{x_1} U^i \partial^2_{x_1} U^i. \tag{3}$$

Higher-order pieces in the effective Lagrangian are not needed for the present calculation.

The systematic perturbative evaluation of the partition function relies on the suppression of loops by some power of momentum. In one spatial dimension, ferromagnetic loops are suppressed by one power of momentum [1]. The corresponding Feynman graphs for the partition function up to order p^5 are depicted in Fig. 1. The leading temperature-dependent contribution stems from the one-loop graph 3 which is of order p^3, as it involves a vertex from $\mathcal{L}^2_{eff}$ (p^2) and one loop (p). The one-loop diagram 5d with an insertion from $\mathcal{L}^4_{eff}$ is of order p^5, as it involves $\mathcal{L}^4_{eff}$ (p^4) and one loop (p). Finally, the two-loop (three-loop) diagrams are of order p^4 (p^5) as they involve one (two) more loops with respect to diagram 3.