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Landau levels in graphene in crossed magnetic and electric fields: Quasi-classical approach

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ABSTRACT

Within the framework of semi-classical approach, the Landau levels in the gapped and gapless graphene in crossed magnetic and electric fields are investigated. We showed that the Landau levels and the distance between them depend on an applied electric field. This is merely due to the non-quadratic electron spectrum and, therefore, it is a unique feature of graphene. At a first approximation, we investigated the effects of electron–electron interaction on the Landau levels. We obtained a general expression for the thermodynamic potential. We concluded that the magnetic properties of graphene can be controlled using the electric field. It is of great practical interest.

1. Introduction

Graphene, a monolayer of carbon atoms, is an ideal structure for studying the physics of two-dimensional systems [1]. The symmetry of the graphene lattice (the hexagonal structure) leads to unique electron spectrum near the Dirac point (K point of the Brillouin zone). Indeed, the energy spectrum consists of two absolutely identical cones (the valence band and the conduction band), which osculate at the Dirac point. This spectrum is stable with respect to the Coulomb interaction [2]. The non-quadratic dependence of the Hamiltonian on the momentum results in a fundamental change in many of the fundamental theories and effects. For example, in Ref. [3], the perturbation theory was constructed for systems with a Hamiltonian, which depends linearly on the momentum. The linear dispersion law of graphene electrons is the reason that in graphene there are qualitatively new effects. For example, the static conductivity of graphene is not zero even at zero carrier concentration [4]. Klein tunneling, known from quantum electrodynamics, is observed in graphene [5]. These and many other unique electronic properties make graphene and graphene-based systems a promising material for different optoelectronic and thermoelectric applications [6–11].

Here we will study the problem of Landau levels in gapped and gapless graphene in crossed magnetic and electric fields. It should be emphasized that graphene exhibits unique properties in magnetic field. For example, Landau levels in graphene are not equidistant [1]. There is zero Landau level. The difference between the first two Landau levels in a magnetic field of 10 T is about 1000 K. This leads to an unusual magneto-transport: fractional quantum Hall effect, which can be observed at room temperature, giant magneto-optical and thermo-magnetic effects etc. Graphene appears as a promising material for magneto-electronics and magneto-optics due to these properties. Obviously, the graphene electrons should behave unusually in crossed magnetic and electric fields. Indeed, yet Lifshitz and Kaganov [12,13] showed that in case with the non-quadratic dependence of electron energy on the momentum, the resonance frequency in crossed magnetic and electric fields depends on the applied electric field. This effect can be used to analyze the forms of isoenergy surfaces in semiconductors [12]. For graphene, the dependence of the Landau levels on the electric field provides an additional opportunity to control the magneto-transport in it. It should be emphasized that such dependence disappears for a quadratic electron dispersion law. In this sense, this effect is another unique feature of graphene.

2. The quasi-classical approach. The Landau levels in gapless graphene

The Landau levels in gapless monolayer graphene in crossed magnetic and electric field were investigated in Ref. [14]. Unfortunately, the authors of Ref. [14] did not refer to the pioneering works of Lifshitz and Kaganov [12,13]. The present work fills the

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gap. Moreover, in contrast to Ref. [14], we will use the quasi-classical approach based on the quantization conditions of Lifshitz-Onsager type [15], as Lifshitz and Kaganov. This condition for the two-dimensional system can be written as (we assume that the plane of electronic system is *XY* and the magnetic field is directed along the *Z* axis)

$$A(\varepsilon)=\frac{2\pi\hbar eH}{c}(n+\gamma_\sigma),\ n=0,1,2,..., \tag{1}$$

where H is the magnetic field strength, $A(\varepsilon)$ is the area enclosed by the electron trajectory at constant energy $\varepsilon(\boldsymbol{p})=\varepsilon$, $\gamma_\sigma=\gamma+1/2(m/m)\sigma$ with $\sigma=\pm1$, $m(\varepsilon_F)=(2\pi)^{-1}dA/d\varepsilon$ is the cyclotron mass, γ is the constant, $\gamma=1/2$ for non-relativistic systems and $\gamma=0$ for graphene. In this study, we neglect the spin splitting of the levels. Function $A(\varepsilon)$ can be found from the following expression [12]

$$A(\varepsilon)=\int d\varepsilon\oint\frac{dl}{v(p)}, \tag{2}$$

where dl is the differential of curve arc, $\varepsilon=$const in momentum space, $v=\sqrt{v_x^2+v_y^2}$.

The tight-binding Hamiltonian for electrons in graphene considering the ability of electrons to hop to the nearest atoms has the form

$$H=-t\sum_{i,j}(a_i^+ b_j+h.c.), \tag{3}$$

where $t\approx2.8\ eV$ is the nearest-neighbor hopping energy between different sublattices, a_i^+ (a_i) annihilates (creates) an electron from sublattice A, $b_{i\sigma}^+, b_{i\sigma}$ are the analogous operators for the sublattice B. The energy spectrum derived from this Hamiltonian has the form

$$\varepsilon(\boldsymbol{q})=\nu_b t|K(\boldsymbol{q})|, \tag{4}$$

$$K(\boldsymbol{q})=\sum_\delta\exp(iq\delta/\hbar)=2\exp(iq_xa/2)\cos\left(\sqrt{3}q_ya/2\right)+\exp(-iq_xa),$$

where $\boldsymbol{q}$ is the two-dimensional wave vector of electrons, $a\approx1.42$ Å is the carbon–carbon distance, δ is the carbon–carbon distance vector, $\nu_b=\pm1$ is the band index, $\nu_b=+1$ corresponds to the conduction band, $\nu_b=-1$ corresponds to the valence band. The energy spectrum near the Dirac point $\boldsymbol{Q}=(2\pi/3a, 2\pi/3\sqrt{3}a)$ has the form

$$\varepsilon(\boldsymbol{k})=\nu_b3ta|\boldsymbol{k}|/2=\nu_bv_F\hbar|\boldsymbol{k}|,$$

where $\boldsymbol{k}=\boldsymbol{q}-\boldsymbol{Q}$, $v_F\approx10^8\ cm/s$ is the Fermi velocity of electrons in graphene. It can be easily shown that for such spectrum the trajectory is circle, i.e. $A(\varepsilon)=\pi\varepsilon^2/v_F^2$. Then we obtain following expression for Landau levels

$$\varepsilon_n=sgn(n)\sqrt{2}v_F\hbar l_H^{-1}\sqrt{|n|}, \tag{5}$$

where $l_H=(\hbar c/eH)^{1/2}$ is the magnetic length. The expression (5) completely coincides with the expression obtained by solving the Dirac equation. It means that the quasi-classical approach is very effective in this case. To find the Landau levels in crossed fields (we assume that the electric field directed along the axis *X*) we can use generalized Lifshitz-Onsager conditions [12]

$$A(\varepsilon^*)=\frac{2\pi\hbar eH}{c}(n+\gamma), \tag{6}$$

where $A(\varepsilon^*)$ is an area enclosed by the electron trajectory at constant energy $\varepsilon^*(\boldsymbol{p})\equiv\varepsilon(\boldsymbol{p})+\boldsymbol{v}_0\boldsymbol{p}=\varepsilon$, where $\boldsymbol{v}_0=c[\boldsymbol{EH}]/H^2$ is the average electron drift velocity, which is directed perpendicularly to $\boldsymbol{E}$ and $\boldsymbol{H}$. Indeed, $\varepsilon^*(\boldsymbol{p})$ is saved in crossed fields (see Appendix). In our case $\boldsymbol{E}\perp\boldsymbol{H}$ and $\boldsymbol{v}_0=cE\boldsymbol{e}_y/H$, where $\boldsymbol{e}_y$ is unit vector along the axis *Y*. Then $\varepsilon^*(\boldsymbol{p})\equiv\varepsilon(\boldsymbol{p})-v_0p_y$. It is easy to show that in this case the curve $\varepsilon^*=$const is an ellipse with the following parameters: $a=\varepsilon/\sqrt{v_F^2-v_0^2}$, $b=\varepsilon v_F/(v_F^2-v_0^2)$, i.e. $A(\varepsilon)=\pi ab=\pi\varepsilon^2v_F/\left(v_F^2-v_0^2\right)^{3/2}$.

For Landau levels, we obtain

$$\varepsilon_n=sgn(n)(1-\beta^2)^{3/4}\sqrt{2}\hbar v_Fl_H^{-1}\sqrt{|n|}, \tag{7}$$

where $\beta=v_0/v_F$. In this paper, we will consider a case when $v_0<v_F$. In reality, the electrons drift perpendicularly to $\boldsymbol{E}$ and $\boldsymbol{H}$. This means that total energy must be written as

$$\varepsilon_{n,p_y}=sgn(n)(1-\beta^2)^{3/4}\sqrt{2}\hbar v_Fl_H^{-1}\sqrt{|n|}+v_0p_y$$

However, in this paper we are interested in only the quantized part of the energy spectrum.

The distance between first two Landau levels can be obtained from Eq. (7)

$$\Delta\varepsilon=(1-\beta^2)^{3/4}\sqrt{2}\hbar v_Fl_H^{-1} \tag{8}$$

i.e. a distance between Landau levels depends on the applied electric field. Such dependence disappears for the quadratic spectrum.

3. The gapped graphene. The electronic spectrum and Landau levels

The equivalence distortion of sublattices in graphene leads to the opening of band gap in the spectrum. The Hamiltonian of the

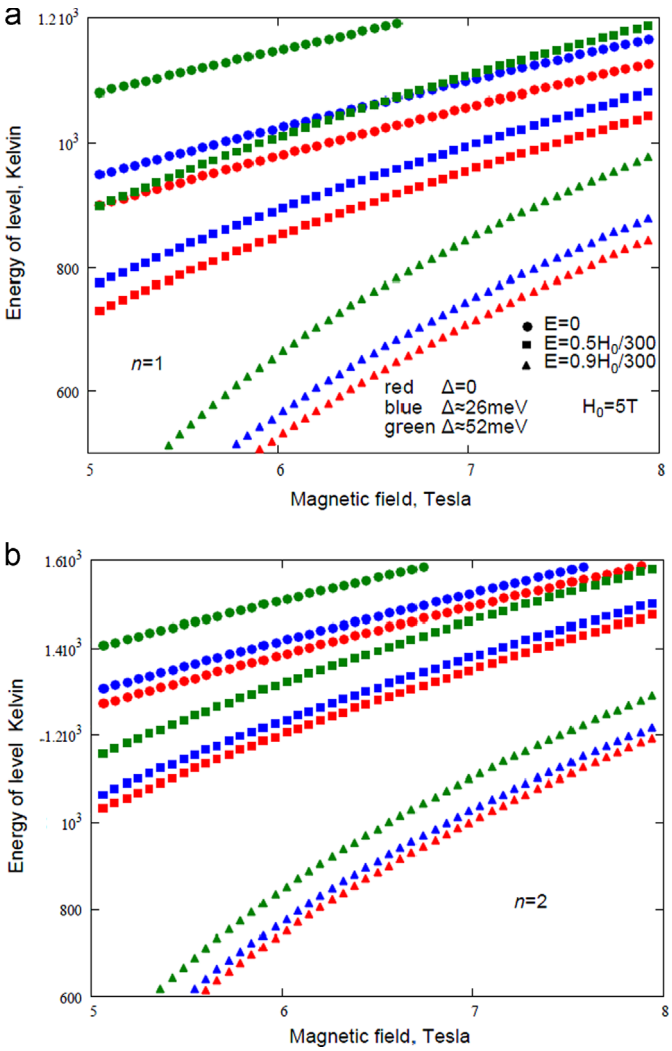

Fig. 1. The Landau levels in graphene in crossed fields for different value of electric field and band gap width. (a) $n=1$ and (b) $n=2$.