Physica B 436 (2014) 10–13

Contents lists available at ScienceDirect

Physica B

journal homepage: www.elsevier.com/locate/physb

Interaction between two-dimensional quantum oscillators and time-dependent forces: Case of a harmonic force

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ARTICLE INFO

Article history:
Received 21 September 2013
Received in revised form
16 November 2013
Accepted 19 November 2013
Available online 26 November 2013

Keywords:
Microwaves
Radiation coupling
Two-dimensional electrons
Zero resistance states

ABSTRACT

We solve analytically the time dependent Schrödinger equation of a two-dimensional quantum oscillator subjected to a time-varying force. We apply the results to the case of a linearly and circularly polarized harmonic force. The main result is that the quantum oscillator orbit center performs a two-dimensional closed loop (elliptical) driven by the force. This theory has been specifically applied to the problem of a two-dimensional electron system subjected to a static and uniform magnetic field and radiation to explain the striking effects of radiation-induced magnetoresistance oscillations and zero resistance states.

1. Introduction

The study of low-dimensional systems is attracting much attention in the recent years. Basic and applied research have focussed on the study of optical and transport properties of these systems. As a result, important and unusual properties have been discovered when, for instance, two-dimensional electron systems (2DESs) are subjected to external AC or DC fields. We can stress microwave-induced resistance oscillations (MIRO) and zero resistance states (ZRS) [1–4]. In this paper we report on a theoretical model which provides an exact solution of the quantum motion of a two-dimensional (2*D*) quantum oscillator (electron with magnetic field) exposed to an arbitrary time-dependent force [5–7]. As a direct application, the case of a harmonic force has been considered. The surprising result is that the guiding center of the quantum oscillator is spatially radiation-driven performing closed loops (elliptical trajectories) in the $x-y$ plane. The results presented in this paper can be applied and generalized to any quantum mechanical oscillator excited by a time-dependent force.

2. Theoretical model

We study a 2DES ($x-y$ plane) under the influence of a static magnetic field aligned in the z-direction and a time-dependent force acting in any direction of such a plane. Considering the symmetric gauge for the vector potential of B ($\overrightarrow{A_B}=-\frac{1}{2}\overrightarrow{r}\times\overrightarrow{B}$), the corresponding Hamiltonian reads

$$H=\frac{P_x^2+P_y^2}{2m}+\frac{w_c}{2}L_z+\frac{1}{2}m\left[\frac{w_c}{2}\right]^2[x^2+y^2]-xF_x(t)-yF_y(t) \qquad (1)$$

where $w_c=eB/m$ is the cyclotron frequency, L_z is the z-component of the electron total angular momentum and F_x and F_y are the components of the time-dependent force. Introducing this Hamiltonian in the time-dependent Schrödinger equation $H\Psi=i\hbar\partial\Psi/\partial t$, we can write

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2\Psi}{\partial x^2}+\frac{\partial^2\Psi}{\partial y^2}\right)+\frac{mw_c^2}{8}(x^2+y^2)\Psi-\frac{i\hbar w_c}{2}\left(x\frac{\partial\Psi}{\partial y}-y\frac{\partial\Psi}{\partial x}\right)$$
$$-[xF_x(t)-yF_y(t)]\Psi=i\hbar\frac{\partial\Psi}{\partial t} \qquad (2)$$

Now we introduce new spatial coordinates, $\xi=x-a(t)$ and $\eta=y-b(t)$, and propose the following solution for the time-dependent Schrödinger equation:

$$\Psi=\phi(\xi,\eta,t)e^{xg_1+yg_2} \qquad (3)$$

where g_1, g_2, a and b are to be determined. The corresponding change of variables $(x,y\rightarrow\xi,\eta)$ has been carried out and the Schrödinger equation transforms into:

$$-\frac{\hbar^2}{2m}\nabla^2_{\xi,\eta}\phi-\frac{i\hbar w_c}{2}\left(\xi\frac{\partial\phi}{\partial\eta}-\eta\frac{\partial\phi}{\partial\xi}\right)+\frac{mw_c^2}{8}(\xi^2+\eta^2)\phi$$
$$-\left(-\frac{\hbar^2 g_1}{m}+b\frac{i\hbar w_c}{2}+i\hbar\dot{a}\right)\frac{\partial\phi}{\partial\xi}+\left(-\frac{\hbar^2 g_2}{m}-a\frac{i\hbar w_c}{2}+i\hbar\dot{b}\right)\frac{\partial\phi}{\partial\eta}$$
$$+\left[\left(\frac{mw_c^2 a}{4}-\frac{i\hbar w_c g_2}{2}-F_x-i\hbar\dot{g}_1\right)\xi+\left(\frac{mw_c^2 b}{4}-\frac{i\hbar w_c g_1}{2}-F_y-i\hbar\dot{g}_2\right)\eta\right.$$

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http://dx.doi.org/10.1016/j.physb.2013.11.033

$$+-\frac{\hbar^2 g_1^2}{2m}-\frac{\hbar^2 g_2^2}{2m}+\frac{mw_c^2a^2}{8}+\frac{mw_c^2b^2}{8}-\frac{i\hbar w_c a g_2}{2}+\frac{i\hbar w_c b g_1}{2}$$
$$-F_x a-F_y b-i\hbar a\dot{g}_1-i\hbar b\dot{g}_2\big]\phi=i\hbar\frac{\partial\phi}{\partial t} \tag{4}$$

Choosing now g_1, g_2, a and b so that the coefficients of $\partial\phi/\partial\xi$, $\partial\phi/\partial\eta$, $\xi\phi$ and $\eta\phi$ vanish

$$-\frac{\hbar^2 g_1}{m}+b\frac{i\hbar w_c}{2}+i\hbar\dot{a}=0 \tag{5}$$

$$-\frac{\hbar^2 g_2}{m}-a\frac{i\hbar w_c}{2}+i\hbar\dot{b}=0 \tag{6}$$

$$\frac{mw_c^2 a}{4}-\frac{i\hbar w_c g_2}{2}-F_x-i\hbar\dot{g}_1=0 \tag{7}$$

$$\frac{mw_c^2 b}{4}-\frac{i\hbar w_c g_1}{2}-F_y-i\hbar\dot{g}_2=0 \tag{8}$$

then the Schrödinger equation can be written as

$$-\frac{\hbar^2}{2m}\nabla^2_{\xi,\eta}\phi-\frac{i\hbar w_c}{2}\left(\xi\frac{\partial\phi}{\partial\eta}-\eta\frac{\partial\phi}{\partial\xi}\right)+\frac{1}{2}m\left(\frac{w_c}{2}\right)^2\left(\xi^2+\eta^2\right)\phi+\lambda(t)\phi=i\hbar\frac{\partial\phi}{\partial t} \tag{9}$$

where

$$\lambda(t)=\frac{1}{2}m\dot{a}^2+\frac{1}{2}m\dot{b}^2+m\frac{w_c}{2}(b\dot{a}-a\dot{b})=\frac{1}{2}m(\dot{a}^2+\dot{b}^2)+\frac{w_c}{2}(bP_a-aP_b) \tag{10}$$

is the classical Lagrangian corresponding to a charged particle in the x–y plane subjected to a uniform and perpendicular magnetic field [8]. After some algebra and using Eqs. (5)–(8) we obtain a system of two coupled equations:

$$m\ddot{a}+2\dot{b}m\frac{w_c}{2}=F_x \tag{11}$$

$$m\ddot{b}+2\dot{a}m\frac{w_c}{2}=F_y \tag{12}$$

In vectorial form, Eqs. (11) and (12) become

$$m(\ddot{a}\vec{i}+\ddot{b}\vec{j})+eB(\dot{b}\vec{i}-\dot{a}\vec{j})=F_x\vec{i}+F_y\vec{j} \tag{13}$$

where

$$eB(\dot{b}\vec{i}-\dot{a}\vec{j})=e\begin{vmatrix}\vec{i},\vec{j},\vec{k}\\ \dot{a},\dot{b},0\\ 0,0,B\end{vmatrix}=e(\vec{v}\times\vec{B}) \tag{14}$$

And finally $m(\ddot{a}\vec{i}+\ddot{b}\vec{j})+e(\vec{v}\times\vec{B})=F_x\vec{i}+F_y\vec{j}$ which is the classical equation of motion of an electron subjected to a perpendicular magnetic field B and an external time-dependent force $(F_x(t)\vec{i}+F_y(t)\vec{j})$. Thus $a(t)$ and $b(t)$ are the corresponding classical solutions of Eq. (14).

The previous Schrödinger equation (Eq. (9)), shows that the spatial variables ξ and η, and t are now separable:

$$\left[-\frac{\hbar^2}{2m}\nabla^2_{\xi,\eta}-\frac{i\hbar w_c}{2}\left(\xi\frac{\partial}{\partial\eta}-\eta\frac{\partial}{\partial\xi}\right)+\frac{1}{2}m\left(\frac{w_c}{2}\right)^2(\xi^2+\eta^2)\right]\phi$$
$$=i\hbar\frac{\partial\phi}{\partial t}-\lambda(t)\phi \tag{15}$$

Now, we propose a solution for ϕ, $\phi=\phi_1(\xi,\eta)\phi_2(t)$, and we can separate the Schrödinger equation into two equations, one depending on spatial variables and the other on time. Therefore, for spatial variables:

$$\left[-\frac{\hbar^2}{2m}\nabla^2_{\xi,\eta}-\frac{i\hbar w_c}{2}\left(\xi\frac{\partial}{\partial\eta}-\eta\frac{\partial}{\partial\xi}\right)+\frac{1}{2}m\left(\frac{w_c}{2}\right)^2\left(\xi^2+\eta^2\right)\right]\phi_1=E_N\phi_1 \tag{16}$$

and for time:

$$i\hbar\frac{\partial\phi_2}{\partial t}-\lambda(t)\phi_2=E_N\phi_2 \tag{17}$$

E_N being a constant that can be identified as the energy.

The solution for the time-dependent equation is straightforward:

$$\phi_2=e^{-i/\hbar\int(\lambda(t)+E_n)\,dt} \tag{18}$$

The spatial-dependent equation can be expressed as

$$(H_{xy}+L_z)\phi_1=E_N\phi_1 \tag{19}$$

where

$$H_{xy}=-\frac{\hbar^2}{2m}\nabla^2_{\xi,\eta}+\frac{1}{2}m\left(\frac{w_c}{2}\right)^2(\xi^2+\eta^2) \tag{20}$$

is the Hamiltonian of an electron in a two-dimensional parabolic confinement caused by B, and

$$L_z=-\frac{i\hbar w_c}{2}\left(\xi\frac{\partial}{\partial\eta}-\eta\frac{\partial}{\partial\xi}\right) \tag{21}$$

These wave functions have analytical expressions and correspond to the well-known Fock–Darwin states [9]. Thus, we can write the solution of the spatial equation in the usual polar coordinates of a Fock–Darwin state (r,θ) as

$$\phi_1=\sqrt{\frac{n!}{2\pi l_B^2 2^{|m|}l_B^{2|m|}(n+|m|)!}}r^{|m|}e^{-im\theta}L_n^{|m|}\left(\frac{r^2}{2l_B^2}\right)e^{-(r^2/4l_B^2)} \tag{22}$$

where n is the radial quantum number, m is the angular momentum quantum number, $L_n^{|m|}$ are the associated Laguerre polynomials and l_B is the effective magnetic length. For the polar coordinates $r^2=\xi^2+\eta^2$ and $re^{i\theta}=\xi+i\eta$. The states energy is given by

$$E_{n,m}=\left(n+\frac{|m|}{2}-\frac{m}{2}+\frac{1}{2}\right)\hbar w_c \tag{23}$$

$$E_{n,m}=\left(N+\frac{1}{2}\right)\hbar w_c \tag{24}$$

Therefore we can write for the wave function:

$$\phi=\phi_1(\xi,\eta)e^{-i/\hbar\int(\lambda(t)+E_n)\,dt}$$
$$=\phi_1\left[x-a(t),y-b(t)\right]e^{-(i/\hbar)\int(\lambda(t)+E_N\ dt)} \tag{25}$$

Once ϕ has been calculated, we can proceed and obtain the final expression for the total wave function:

$$\Psi(x,y,t)=\phi\left[(x-a(t)),(y-b(t)),t\right]$$
$$\times e^{(i/\hbar)\left[m^*((da/dt)x+(db/dt)y)+m^*w_c(bx-ay)/2-\int_0^t\lambda(t')\,dt'\right]} \tag{26}$$

This is the exact form of the analytical solution of a quantum oscillator under the influence of time-varying force (two-dimensional electron system under a static magnetic field and a time dependent force).

3. Results

The main result is that, apart from phase factors, the wave function Ψ is the same as a Fock–Darwin state where the center of the Larmor orbits is displaced by $a(t)$ in the x-direction and $b(t)$ in the y-direction. The magnitude and nature of this displacement will depend on the type of the time-dependent force. If the time dependent force is harmonic (radiation), several cases can be considered depending on the orientation of the force components inside the 2D plane.

If we consider that the harmonic time-dependent force $F(t)$ is oscillating in one plane (linearly polarized), this plane can be in different polarization angles (α) regarding the transport direction,