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# Fast analysis of ferromagnetic shields by means of fixed point iterative technique

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ABSTRACT

In this paper we propose a fast iterative method for the analysis of ferromagnetic shields taking into account the nonlinearity of the magnetic characteristic. This novel iteration scheme handles the nonlinear *B–H* characteristic by means of the Fixed Point technique. Moreover, the convolution properties of the field problem are exploited by means of the FFT operator. A case study is presented to demonstrate the performance of the proposed method.



## 1. Introduction

This paper focuses on the analysis of ferromagnetic thin shields taking into account the nonlinear characteristic of the magnetic material. Several numerical techniques are described in the literature, such as FEM [1] or FEM/BEM [2], but some drawbacks could arise when dealing with thin slabs of ferromagnetic materials. For instance, standard FEM can lead to a mesh with high number of unknowns and numerical instabilities can occur [1]. Moreover, a linear approximation of the magnetic characteristic is often assumed but, in case of strong source field or magnetic shields close to the source, the magnetic flux density inside the shield could reach saturation and therefore the planning/design of the shield should take into account the nonlinearity of the material. In this paper we propose a method that handle the nonlinearity of the material by means of the Fixed Point (FP) technique [3] and, at the same time, exploits the convolution properties of the field problem by means of the FFT operator [4]. In the following section the theoretical background is explained. In Section 3 the *Fast Fixed Point* (Fast FP) technique is introduced. In Section 4, the effectiveness of the proposed algorithm is compared with the Standard FP technique [5]. Finally, in the same section, the proposed formulation is applied to the case of field mitigation of a 1250 kVA transformer.

## 2. Theoretical background

The magnetic field $\mathbf{H}$ inside the wall of the magnetic shell can be written as

$$\mathbf{H} = \mathbf{H}_\mathrm{s} + \mathbf{H}_\mathrm{m}, \tag{1}$$

where $\mathbf{H}_\mathrm{s}$ is the applied magnetic field generated by the external sources and $\mathbf{H}_\mathrm{m}$ is the magnetostatic field contributed by the magnetization $\mathbf{M}$ inside the magnetized wall. The numerical computation of the magnetostatic field term $\mathbf{H}_\mathrm{m}$ can be obtained, following [6], by discretizing the shell volume into rectangular blocks and representing the magnetization by a discrete distribution of fields $\mathbf{M}_i$ at points $\mathbf{r}_i$. Here, $\mathbf{M}$ is the corresponding averaged value on the $i$th element. The magnetostatic field at $\mathbf{r}_i$ is then

$$\mathbf{H}_{\mathrm{m}i} = -\sum_j N(\mathbf{r}_i - \mathbf{r}_j)\mathbf{M}_j, \tag{2}$$

where $N$ is the demagnetizing tensor [6–8] and $\mathbf{H}_{\mathrm{m}i}$ is the averaged magnetostatic field on the $i$th element. The magnetization $\mathbf{M}_i$ is related to the magnetic field $\mathbf{H}_i$ through the nonlinear characteristic of the material that, in the following, is approximated by the single-valued normal curve $\mathbf{M}_i = g(|\mathbf{H}_i|)\cdot\mathbf{H}_i/|\mathbf{H}_i| = \mathbf{g}(\mathbf{H}_i)$. Introducing this relation in Eq. (1) and taking into account Eq. (2) the following system of nonlinear equations is obtained

$$g^{-1}(|\mathbf{M}_i|)\cdot\mathbf{M}_i/|\mathbf{M}_i| = -\sum_j N(\mathbf{r}_i - \mathbf{r}_j)\mathbf{M}_j + \mathbf{H}_{\mathrm{s}i}, \tag{3}$$

where $g^{-1}(\cdot)$ is the inverse magnetic characteristic. The unknown field $\mathbf{M}_i$ can be obtained by solving the nonlinear system (3). This nonlinear problem can be effectively handled by means of the Fixed Point (FP) technique [3]. By this method, the nonlinear relation $g^{-1}(\cdot)$ is splitted into a linear part and a residual term

$$\mathbf{H}_i = \mathbf{M}_i/\chi_{\mathrm{FP}} + \mathbf{H}_{\mathrm{res}i}, \tag{4}$$

where $\mathbf{H}_i$ and $\mathbf{M}_i$ are the magnetic field and the magnetization at point $\mathbf{r}_i$ respectively, the residual $\mathbf{H}_{\mathrm{res}i}$ is a local nonlinear function of the magnetic field $\mathbf{H}_i$, and $\chi_{\mathrm{FP}}$ is a constant whose value is discussed in Section 3. By replacing Eqs. (2) and (4) in Eq. (1) we

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obtain

$$\mathbf{M}_i/\chi_{\rm FP}+\sum_j N(\mathbf{r}_i-\mathbf{r}_j)\mathbf{M}_j=\mathbf{H}_{\rm si}-\mathbf{H}_{\rm resi} \tag{5}$$

Eq. (3) results now splitted into the two equations (4) and (5), whose simultaneous solution provides the unknown fields. The advantages of this formulation are inherent in the possibility of solving the two equations iteratively [3] as explained in the next section.

## 3. Fast iterative technique

The iterative solution of Eqs. (4) and (5) can be achieved, in principle, through the following algorithm (standard Fixed Point technique). Known the residual field from iteration $k$th, the following equation is obtained from Eq. (5):

$$\mathbf{M}_i^{k+1}/\chi_{\rm FP}+\sum_j N(\mathbf{r}_i-\mathbf{r}_j)\mathbf{M}_j^{k+1}=\mathbf{H}_{\rm si}-\mathbf{H}_{\rm resi}^k, \tag{6}$$

where $\mathbf{H}_{\rm resi}^k$ is the (known) residual magnetic field at iteration $k$th and $\mathbf{M}_i^{k+1}$ is the (unknown) magnetization field. Afterwards, the magnetic field is calculated through Eq. (4)

$$\mathbf{H}_i^{k+1}=\mathbf{M}_i^{k+1}/\chi_{\rm FP}+\mathbf{H}_{\rm resi}^k. \tag{7}$$

Finally, the residual field is updated

$$\mathbf{H}_{\rm resi}^{k+1}=\mathbf{H}_i^{k+1}-\mathbf{g}(\mathbf{H}_i^{k+1})/\chi_{\rm FP}. \tag{8}$$

It is worth noting that the application of this algorithm makes use of the direct $g(\cdot)$ function between $\mathbf{H}$ and $\mathbf{M}$.

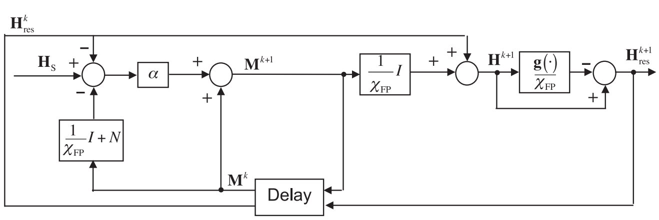


**Fig. 1.** Block diagram of the Fast FP algorithm. It summarizes Eqs. (10), (11), (7), and (8) that are repeated until the error norm $E^{k+1}=\|\mathbf{H}_{\rm res}^{k+1}-\mathbf{H}_{\rm res}^k\|_2$ becomes sufficiently small. Note that, for the sake of simplicity, the subscript $i$ is omitted.

This algorithm converges to the Fixed Point, i.e. the solution of (4) and (5), starting from whatever residual field, on condition that the constant $\chi_{\rm FP}$ is suitably chosen. Following Ref. [3] its optimal value is given by

$$\chi_{\rm FP}=(\chi_{\min}+\chi_{\max})/2, \tag{9}$$

where $\chi_{\min}$ and $\chi_{\max}$ are the minimum and maximum slopes of the magnetic characteristic $g(\cdot)$ respectively. In any case, $\chi_{\rm FP}>\chi_{\max}/2$ is needed to ensure convergence [3]. However, the solution of the system (6) (at each FP iterative step) can became very time consuming because the coefficient matrix is full. This problem can be solved by the application of the iterative method proposed in Ref. [4] for the case of linear constitutive relation. Hence, the iterative Fixed Point algorithm and the iterative method of Ref. [4] can be eventually combined as explained in the following.

Known the residual field from iteration $k$th, Eq. (6) is replaced, according to Ref. [4], by

$$\mathbf{M}_i^{k+1}=\mathbf{M}_i^k+\alpha\cdot[\mathbf{H}_{si}-\mathbf{H}_{\rm resi}^k-(\mathbf{M}_i^k/\chi_{\rm FP}-\mathbf{H}_{\rm mi}^k)], \tag{10}$$

where $i$th index refers to a rectangular block element, $\mathbf{M}_i^k$ is the magnetization distribution at the $k$th iteration, $\mathbf{H}_{\rm mi}^k$ is the magnetostatic field associated to magnetization $\mathbf{M}_i^k$, and $\alpha$ is the relaxation parameter discussed in Ref. [4]. The magnetostatic field is computed by the formula

$$\mathbf{H}_{\rm mi}^k=-\sum_j N(\mathbf{r}_i-\mathbf{r}_j)\mathbf{M}_j^k, \tag{11}$$

where the convolution product of Eq. (11) can be computed very fast by using the FFT algorithm, whose complexity is $O(n \log n)$. The residual is then updated by using Eqs. (7) and (8).

The steps (10), (11), (7), and (8) are then repeated until the error norm $E^{k+1}=\|\mathbf{H}_{\rm res}^{k+1}-\mathbf{H}_{\rm res}^k\|_2$ becomes sufficiently small. This algorithm (fast FP) converges to the solution for any initial condition ($\mathbf{M}_i^0$, $\mathbf{H}_{\rm resi}^0$) provided that the constant $\chi_{\rm FP}$ is suitably computed according to Eq. (9) and the relaxation coefficient is chosen according to the analysis reported in Ref. [4]. The whole iterative procedure is diagrammatically explained in Fig. 1.

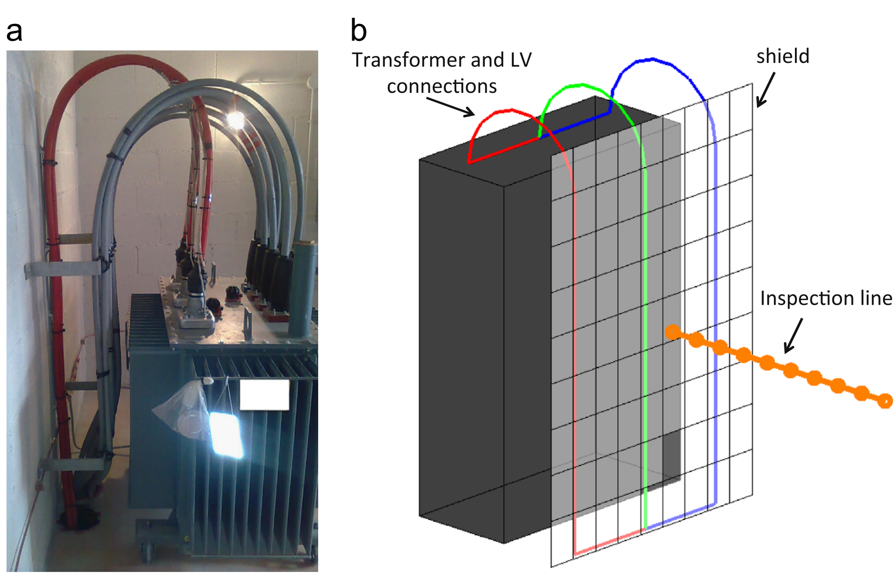


**Fig. 2.** A 1250 kVA transformer is selected as a case study (a). The mitigation is provided by a metallic slab installed on the wall. The shield performance is evaluated along the inspection line (b).