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Defect solitons in saturable nonlinearity media with parity-time symmetric optical lattices

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ABSTRACT

We reported the existence and stability of defect solitons in saturable nonlinearity media with parity-time (PT) symmetric optical lattices. Families of fundamental and dipole solitons are found in the semi-infinite gap and the first gap. The power of solitons increases with the increasing of the propagation constant and saturation parameter. The existence areas of fundamental and dipole solitons shrink with the growth of saturation parameter. The instability of dipole solitons for positive and no defect induced by the imaginary part of PT symmetric potentials can be suppressed by the saturation nonlinearity, but for negative defect it cannot be suppressed by the saturation nonlinearity.

1. Introduction

In 1998, Bender et al. found that a wide class of non-Hermitian Hamiltonians can actually possess entirely real spectra as long as they respect parity-time (PT) symmetry [1]. The concept was introduced to optics by many people [2,4–6]. A necessary condition for a Hamiltonian to be PT symmetry is that its complex potential satisfies $V(x)=V^{*}(-x)$ [7,8]. This implies that the real part of the potential must be an even function of position and that the imaginary part must be odd. In optics, PT-symmetric structures can be constructed by inclusion of gain or loss regions into waveguides, which make the complex refractive-index distribution obeying the condition $n(x)=n^{*}(-x)$ [3–6]. Experimental realizations of such PT systems have been reported recently. Guo et al. have observed passive PT-symmetry breaking and phase transition that lead to a loss-induced optical transparency in specially designed pseudo-Hermitian guiding potentials [9]. Rüter et al. have observed the spontaneous PT symmetry breaking and power oscillations violating left-right symmetry in PT optical coupled linear system involving a complex index potential [10]. PT symmetries have been realized in the LRC circuits [11,12], and dual behavior of PT-symmetric scattering has been observed [13].

Recently, Regensburger et al. represent the application of PT-symmetry to a new generation of multifunctional optical devices and networks experimentally [14]. Those theoretical and experimental results led to the proposal of a new class of PT-symmetric synthetic materials with intriguing and unexpected properties [10,14,15]. In optics, nonlinearities in the PT-symmetric systems have been considered by many authors [15–19], especially in the PT-symmetric optical lattices [5,6,15], and some new kinds of soliton were found and investigated [5,20–23].

Defect solitons in optical lattices with specially designed defect have attracted special attention due to their novel and unique characteristics in diverse areas of physics and have been applied extensively for steering of optical beams [24–27], all-optical switches [28,29], filtering [30] and routing of optical signals [31]. Defect solitons in local Kerr nonlinearity media with PT symmetric optical lattices have been studied, and stable solitons are found mainly in the semi-infinite gap [22]. Recently, we have studied defect solitons in nonlocal Kerr nonlinearity media with PT symmetric optical lattices and found that the nonlocal nonlinearity can expand stable ranges of solitons [23]. However, the optical properties with PT symmetric potentials in saturable nonlinearity media have not been studied. It is noteworthy that the nonlinearity in the photorefractive media, in which Rüter et al. have observed the non-reciprocal wave propagation [10], is saturable nonlinearity. The nonlinearity saturation suppresses the collapse of fundamental solitons in two and three dimensions [32,33], which opens the door for their experimental observation in multidimensional optical beams. The instability of higher-order (multihump) solitons is not suppressed by the nonlinearity saturation in general lattices [34–36]. However, in this paper, we find

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that the instability of dipole solitons can be suppressed by the saturation nonlinearity in the PT symmetric optical lattices. The saturable nonlinearity can expand stable ranges of fundamental and dipole solitons, especially in the first gap. It is found that the stability of defect solitons depends on the defect, the degree of saturable nonlinearity, PT potential and the symmetry of solitons.

2. Theoretical model

We consider the propagation of light beam in PT symmetric defective lattices embedded into a focusing saturable medium. The evolution of complex normalized amplitude U of the light field can be described by the following nonlinear Schrödinger equation:

$$i\frac{\partial U}{\partial z}+\frac{1}{2}\frac{\partial^2 U}{\partial x^2}+p[V(x)+iW(x)]U+\frac{U|U|^2}{1+s|U|^2}=0, \tag{1}$$

where the transverse x and longitudinal z coordinates are normalized to the width and diffraction; p is the depth of PT symmetric potentials; s stands for the degree of saturable nonlinearity; according to Ref. [37], s is depended on the light-induced maximum refractive-index change and s is positive. $V(x)$ and $W(x)$ are the real and imaginary parts of PT symmetric defective potentials, respectively, which are assumed in this paper as

$$V(x)=\cos^2(x)[1+\epsilon\exp(-x^8/128)],\quad W(x)=W_0\sin(2x), \tag{2}$$

here ϵ represents the strength of the defect, and defect is expressed as a super-Gaussian profile [22]. $\epsilon=0$ corresponds to uniform lattice, and the soliton in this lattice is a gap soliton, so we define this lattice as no defect. When $\epsilon>0$, the center refractive index is greater than that of both sides, and the defect is defined as positive defect. When $\epsilon<0$, the center refractive index is lower than that of both sides, and the defect is defined as negative defect. The parameter W_0 represents the strength of the imaginary part compared with the real part. The linearized normalization relation of Eq. (1) is given in Ref. [4].

The linearized version in Eq. (1) has a Bloch band structure when $\epsilon=0$. The band diagram can be entirely real when the system is operated below the phase transition point ($W_0=0.5$) [5]. In this paper, only PT lattice with its Bloch spectrum below the phase transition point is considered, and without loss of generality, parameters $W_0=0.15$ and $W_0=0.25$ are adopted throughout the paper. Typical PT lattice profile and its Bloch band structure with $W_0=0.15$ are shown in Fig. 1(a) and (b), respectively. From Fig. 1(b), we can see that the region of semi-infinite gap for $p=4$ and $W_0=0.15$ is $b>2.7$, and the first and the second gaps locate in $0.83<b<2.63$ and $-0.55<b<0.15$, respectively, where b is the propagation constant.

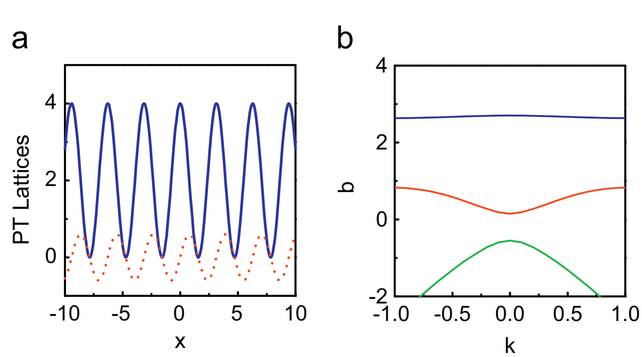

Fig. 1. For $p=4$ and $W_0=0.15$, (a) profile of the PT lattice (blue and red lines represent the real and imaginary parts, respectively) and (b) band structure of the lattice in (a). (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this article.)

We search for stationary solutions to Eq. (1) in the form $U=f(x)\exp(ibz)$, where $f(x)$ is the complex function satisfies equations:

$$bf=\frac{1}{2}\frac{\partial^2 f}{\partial x^2}+p[V(x)+iW(x)]f+\frac{f|f|^2}{1+s|f|^2}. \tag{3}$$

The solutions of defect solitons are gotten numerically from Eq. (3). Families of solitons are determined by the propagation constant b, saturation parameter s, lattice depth p, and the strength of the imaginary part of PT symmetric potential W_0. Without loss of generality, we fixed lattice depth $p=4$ and varied b, s, W_0 throughout the paper unless stated otherwise. To elucidate the stability of defect solitons, we search for the perturbed solution to Eq. (1) in the form $U(x,z)=[f(x)+u(x,z)+iv(x,z)]\exp(ibz)$, where the real $[u(x,z)]$ and imaginary $[v(x,z)]$ parts of the perturbation can grow with a complex rate δ upon propagation, i.e. $u(z,x)=p_1(x)e^{\delta z}$ and $v(z,x)=p_2(x)e^{\delta z}$, respectively. Linearization of Eq. (1) around the stationary solution $f(x)$ yields the eigenvalue problem

$$\begin{aligned}\delta v=&\frac{1}{2}\frac{\partial^2 u}{\partial x^2}-bu+p(Vu-Wv)\\&+\frac{\{[\mathrm{Re}(f)]^2-[\mathrm{Im}(f)]^2+2|f|^2\}u+2\,\mathrm{Re}(f)\,\mathrm{Im}(f)v}{(1+s|f|^2)}\\&-\frac{2s|f|^2\,\mathrm{Re}(f)[\mathrm{Re}(f)u+\mathrm{Im}(f)v]}{(1+s|f|^2)^2},\end{aligned} \tag{4}$$

$$\begin{aligned}\delta u=&-\frac{1}{2}\frac{\partial^2 v}{\partial x^2}+bv-p(Vu+Wv)\\&-\frac{\{[\mathrm{Im}(f)]^2-[\mathrm{Re}(f)]^2+2|f|^2\}v+2\,\mathrm{Re}(f)\,\mathrm{Im}(f)u}{(1+s|f|)}\\&+\frac{2s|f|^2\,\mathrm{Im}(f)[\mathrm{Re}(f)u+\mathrm{Im}(f)v]}{(1+s|f|^2)^2}.\end{aligned} \tag{5}$$

The above coupled equations can be solved numerically to find the maximum value of $\mathrm{Re}(\delta)$. If $\mathrm{Re}(\delta)>0$, solitons are unstable. Otherwise, they are stable.

3. Defect solitons

In the saturable nonlinearity media with PT symmetric defective lattices, we find two types of defect solitons for positive, no, and negative defect cases, as shown in Figs. 2 and 3. The first type is nodeless fundamental solitons, which can exist stably in the semi-infinite gap. We consider that fundamental solitons are PT symmetric, because their real parts are even and imaginary parts are odd, that is similar to the PT symmetric potential. The other type of defect solitons, which exists in the first gap, is called dipole solitons in this paper, because they have two significant intensity peaks. The real parts of dipole solitons are odd and the imaginary parts are even, that is opposite to the PT symmetric potentials and fundamental solitons. In this paper we state that dipole solitons are PT antisymmetric.

For positive, no, and negative defect, we take $\epsilon=0.5$, $\epsilon=0$ and $\epsilon=-0.5$, respectively. The field profiles of the defect solitons are shown in Fig. 2(a)–(d) for different defects, saturation parameter s and symmetric. Fig. 2(a) shows fundamental soliton in the semi-infinite gap, while dipole soliton in the first gap is shown in Fig. 2 (b) for positive defects. One can see that the poles of dipole solitons are located inside the central channel of the lattice. The properties of defect solitons with zero defects are similar to those with positive defects. Differing from the positive and zero defects, fundamental solitons can exist in both the semi-infinite gap and the first gap, as shown in Fig. 2(c) and (d).